The Turducken Cooking Challenge

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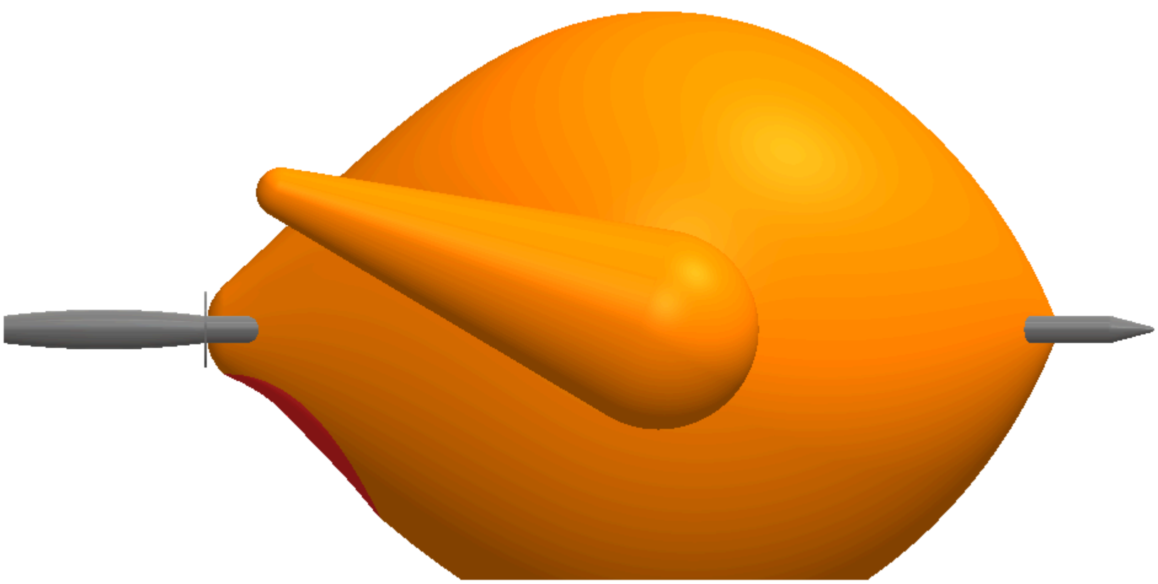

ME 408 Professor Bondi

Computer Aided Engineering Fall 2017

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Introduction

In an effort to aid families who cook turkeys for Thanksgiving, a new method of cooking large turduckens in a conventional oven has been created. This method uses a set of two electric resistance skewers to cook the inside of a turducken as the surface is cooked by the oven; these skewers reduce cooking time, minimize burning, and maintain a roughly consistent temperature in the birds as they cook. This design project focuses on the use of finite element methods and hand calculations to properly design and place the skewers within the turducken. To be considered fully cooked, every part of the turducken must be at least 185°F; a part of the turducken is considered burned if this part exceeds 265°F. The skewers to be designed must have a maximum cross-sectional area of 0.25 in^2 and must not exceed 0.75 in. in any dimension of the cross section. The length of the skewers, the portions of these skewers that are heated, and the heat output of each skewer are unrestricted. Additionally, the skewers can be switched on or off once during the cooking procedure. The oven the turducken is placed inside is allowed to operate between 350°F and 500°F; the temperature setting of the oven can also be adjusted once during the cooking procedure. Below is a section view of the scanned turducken provided by the client.

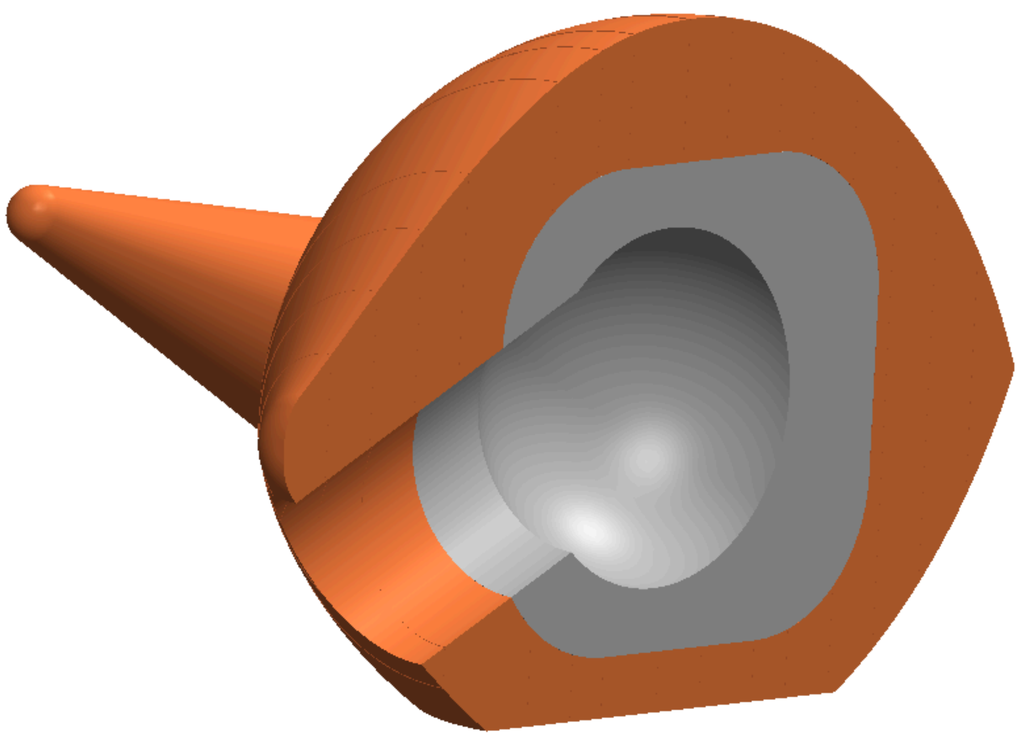

Figure 1: The cross-section of a scanned turducken as provided by the client

Design Assumptions

The given parameters, constraints, and design assumptions are below:

- The turducken begins cooking after being fully refrigerated at 42°F.
  - This is a reasonable assumption, assuming the turducken is stored in a refrigerator long enough for the entire turducken to cool to 42°F uniformly.
- The turducken is fully cooked when the minimum temperature in the turducken is at least 185°F, and an area of the turducken is burned if it is at least 265°F.
- The ducken and stuffing are homogeneous mixtures whose thermal properties are weighted according to their relative weights in the mixture.
  - Assuming that the ducken and stuffing are mixed together extremely well, this assumption is justified.
- The skewers can change settings instantaneously.
  - Given that the skewers use resistance heating and that power could be put into or cut from the skewers at any point in time, this assumption is justified.
- The skewer's power source is not considered (within reason), the skewer is manufacturable.
  - Detailed skewer designs are beyond the scope of this project. As long as realistic specifications for the skewer are chosen, it is assumed that the skewer could be manufactured.
- The stuffing (in addition to the turkey and ducken) must be heated to at least 185°F to be considered fully cooked.
  - Given that the stuffing is in contact with both the ducken and the turkey, the stuffing should also be heated to prevent undercooked food contamination.
- The stuffing, ducken, and turkey are assumed to be isotropic and homogeneous.
  - In reality, poultry has orthotropic thermal properties due to the meat being mostly lean muscle. Since the ducken is a homogenous mixture of duck and chicken, this assumption is reasonable, as the muscle strands could have been shredded in a powerful blender.
- The density, thermal conductivity, and specific heat only change appreciably at 212°F and 265°F.
  - As will be further explained in the Thermal Properties section, the thermal properties of food are shown to not vary significantly with temperature and can be considered constant until 212°F, where all of the moisture in the food evaporates, and 265°F, where it is assumed the food turns fully into ash.

Stuffing Recipe

The stuffing is to be placed into the entire cavity of the turducken. In order to aid cooking, a stuffing with a high heat conductivity is needed to carry heat from the skewers to the ducken as quickly as possible. A stuffing with a high heat capacity is also desired in order to hold residual heat after the skewers are turned off in order to store as much heat energy as possible per unit temperature.

The stuffing is composed of 75% mashed potatoes and 25% cranberries. The mixture, as with the ducken, is a homogeneous mixture of cranberries and mashed potatoes with a weighted average density, thermal conductivity, and heat capacity calculated based on the amount of cranberries and mashed potatoes in the mixture. See the Thermal Properties section and Appendix A for more detailed calculations. The benefit of this choice is that both components have a naturally high thermal conductivity and heat capacity, and that the combination is both traditional and relatively tasty when combined with the ducken and turkey.

Potatoes and cranberries have the benefits of high capacitance and high thermal conductivity due to their large water contents of around 80%. Another benefit of the largely potato-based stuffing is its feasibility as a stuffing. The goal of the project is to make lives easy: potatoes are cheap, easy to handle, have no gluten or lactose, and mashed potatoes are incredibly malleable and can be shaped to hold any form, especially when baked. Additionally, cranberries can be easily purchased around Thanksgiving, making this food combination both delicious and easy to access. Below is a more formal stuffing recipe as it is to be used in cooking (see Appendix C for cooking instructions):

The Perfect Turducken Stuffing Recipe:

3 cup potatoes - cubed (usually about 3 potatoes)

1 cup fresh cranberries

Salt and pepper to taste

Place potatoes and cranberries in food processor. Pulse until thoroughly mixed, a light pink to pale red color.

Place stuffing in turducken cavity with clean hands. Pack tight to remove most of the air.

Thermal Properties

To simulate the cooking time of the turducken, material properties have to be determined for the stuffing, ducken, and turkey. ANSYS requires the following three thermal properties to fully characterize a material: specific heat, thermal conductivity, and density. These properties are calculated using a procedure outlined in the 2006 American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) Handbook (1). According to the handbook, the thermal properties of foods can be calculated by taking a weighted average of the thermal properties of six common food constituents: water, protein, fat, carbohydrate, fiber, and ash. Below are the thermal properties of the six common food constituents.

Table 1 Thermal Property Models for Food Components ($-40 \le t \le 300°F$)

Thermal Property	Food Component	Thermal Property Model
Thermal conductivity, Btu/(h·ft·°F)	Protein	$k = 9.0535 \times 10^{-2} + 4.1486 \times 10^{-4}t - 4.8467 \times 10^{-7}t^2$
	Fat	$k = 1.0722 \times 10^{-1} - 8.6581 \times 10^{-5}t - 3.1652 \times 10^{-8}t^2$
	Carbohydrate	$k = 1.0133 \times 10^{-1} + 4.9478 \times 10^{-4}t - 7.7238 \times 10^{-7}t^2$
	Fiber	$k = 9.2499 \times 10^{-2} + 4.3731 \times 10^{-4}t - 5.6500 \times 10^{-7}t^2$
	Ash	$k = 1.7553 \times 10^{-1} + 4.8292 \times 10^{-4}t - 5.1839 \times 10^{-7}t^2$
Density, lb/ft^3	Protein	$\rho = 8.3599 \times 10^{1} - 1.7979 \times 10^{-2}t$
	Fat	$\rho = 5.8246 \times 10^{1} - 1.4482 \times 10^{-2}t$
	Carbohydrate	$\rho = 1.0017 \times 10^{2} - 1.0767 \times 10^{-2}t$
	Fiber	$\rho = 8.2280 \times 10^{1} - 1.2690 \times 10^{-2}t$
	Ash	$\rho = 1.5162 \times 10^{2} - 9.7329 \times 10^{-3}t$
Specific heat, Btu/(lb·°F)	Protein	$c_p = 4.7442 \times 10^{-1} + 1.6661 \times 10^{-4}t - 9.6784 \times 10^{-8}t^2$
	Fat	$c_p = 4.6730 \times 10^{-1} + 2.1815 \times 10^{-4}t - 3.5391 \times 10^{-7}t^2$
	Carbohydrate	$c_p = 3.6114 \times 10^{-1} + 2.8843 \times 10^{-4}t - 4.3788 \times 10^{-7}t^2$
	Fiber	$c_p = 4.3276 \times 10^{-1} + 2.6485 \times 10^{-4}t - 3.4285 \times 10^{-7}t^2$
	Ash	$c_p = 2.5266 \times 10^{-1} + 2.6810 \times 10^{-4}t - 2.7141 \times 10^{-7}t^2$

Source: Choi and Okos (1986)

Figure 2: ASHRAE Handbook thermal properties as a function of temperature for the six common food constituents

According to the ASHRAE Handbook, the specific heat (c) of a food is calculated by taking a weighted average of the specific heat of each of the six food constituents weighed by mass fraction. This can be written as:

$$c = \sum_i c_i x_i,$$

where c_i is the specific heat of each constituent and x_i is the mass fraction of each constituent. For calculating the thermal conductivity (k), the approach is similar, replacing the mass fraction x_i with the volume fraction x_i^v and including the constituent's density ρ_i:

$$k = \sum_i x_i^v k_i \text{ where } x_i^v = \frac{x_i/\rho_i}{\sum_i x_i/\rho_i}$$

Lastly, the density of a food is calculated as follows:

$$\rho = \frac{1}{\sum_i x_i/\rho_i}$$

The handbook provides mass percentages of the food constituents in unfrozen foods, allowing the initial thermal properties for all ingredients to be calculated. As the ducken is a homogenous mixture of equal parts duck and chicken, thermal properties of the ducken are equal to the average of the properties of duck and chicken. Lastly, the thermal properties of the stuffing are a weighted average of the thermal properties of potatoes and stuffing. See Appendix A for detailed calculations and the thermal properties used for each ingredient.

The 2006 ASHRAE Handbook lists the mass percentages of the six food constituents for unfrozen food. These properties are accurate for food that has just been removed from a refrigerator up to a certain temperature. The thermal properties of all the food constituents vary with temperature; however, the variation is two or more orders of magnitude smaller than the magnitude of the thermal property. Therefore, this variation is considered negligible. However, thermal properties experience a significant change for food that has reached 212°F, the boiling point of water. At this temperature, the thermal properties of unfrozen food are no longer accurate, as the water in the food begins to evaporate. In order to account for this, thermal properties of each ingredient are recalculated at 212°F with the mass percentage of water set to zero. This step change in material properties does not occur in reality, as a portion of the food will remain at a constant 212°F while all the water boils off. However, it is assumed that each element of the mesh is sufficiently small such that the time required for water to boil off is negligible compared to the overall cook time.

The ingredients of the turducken experience one final change in thermal properties once they reach 265°F. At this temperature, all ingredients are considered burned. As a portion of the turducken (including the stuffing) reaches this temperature, it is assumed that the organic compounds that region break down: proteins, fats, carbohydrates, and fibers would break down under that high of a temperature. After these materials break down, the only remaining compounds are inorganic. The ASHRAE Handbook groups these compounds under the food constituent *ash*. As a result, after 265°F a region of burned food is considered to be completely ash. See Appendix A for a table denoting the step changes in density, conductivity, and specific heat for the stuffing, ducken, and turkey. Below is an example of a table created in ANSYS that denoted the changes in material properties with respect to temperature.

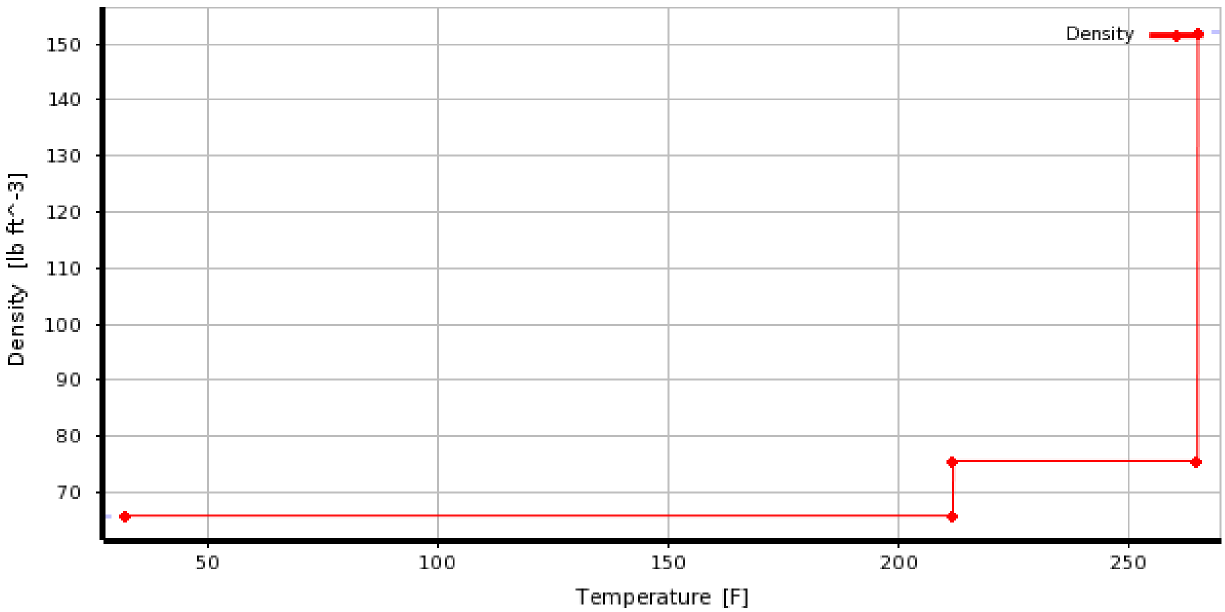

Figure 3: An example of the step changes in properties described above (density of turkey is shown)

Hand Calculations: Setup

In an effort to preserve the relative spherical symmetry of the turducken model, the hand calculation approximates half the turducken as a set of concentric spheres. Nodes are placed along the surfaces of these spheres, extending radially outward from the center of the innermost sphere to the surface of the outermost sphere. To do so, the actual volume of half the stuffing, ducken, and turkey is found using SolidWorks and set equal to the volume of a sphere with an equivalent radius:

$$V_{actual} = (\tfrac{4}{3})\pi r_{eq}^{\ 3},$$

where V_{actual} is the measured volume of half the stuffing, half the ducken, or half the turkey, and r_{eq} is the equivalent radius of a sphere used in the hand calculation model. See Appendix B for the values of these volumes. Below is a diagram of a cross section of the concentric sphere model.

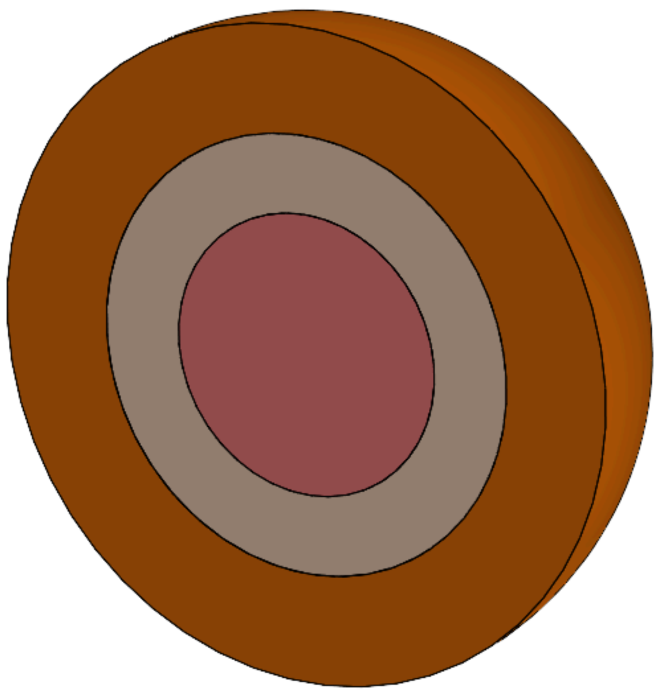

Figure 4: Cross section of the concentric sphere model

In this diagram, the red sphere represents the stuffing, the gray sphere represents the ducken, and the brown sphere represents the turkey. The nodes placed on the surfaces of these spheres measure the temperature at these node locations. To calculate temperatures along the spheres, the heat transfer process is modeled as an equivalent circuit. Using the densities, thermal conductivities, and heat capacities calculated in the Thermal Properties section, a circuit with equivalent resistors, current generators, and capacitors is created. In this case, the thermal conductivities and convection can be modeled as resistors whose magnitudes vary with distance and surface area. The heat capacities can be modeled as capacitors whose magnitude varies with density and volume. The skewer can be modeled as a current generator attached to a node corresponding to the skewer's location in the turducken. The temperature differences can be modeled as changes in voltage, completing the analogy. This model consists of 7 resistors, 3 capacitors, 1 current generator, and 6 nodes to measure temperatures, with the current generator (Q_{dot}) attached to the first node (T_1). There is only one current generator because only half of the turducken is modeled in this way. Six nodes are inside the turducken, and the rightmost node (T_∞) is the temperature of the oven. The magnitude of Q_{dot} that had been chosen is 0.05213 $\frac{BTU}{s}$, and the magnitude of the temperature of the oven (T_∞) that had been chosen is 350ºF. These magnitudes, as well as other numerical results, are all tabulated in Appendix B. Below is the circuit as stated.

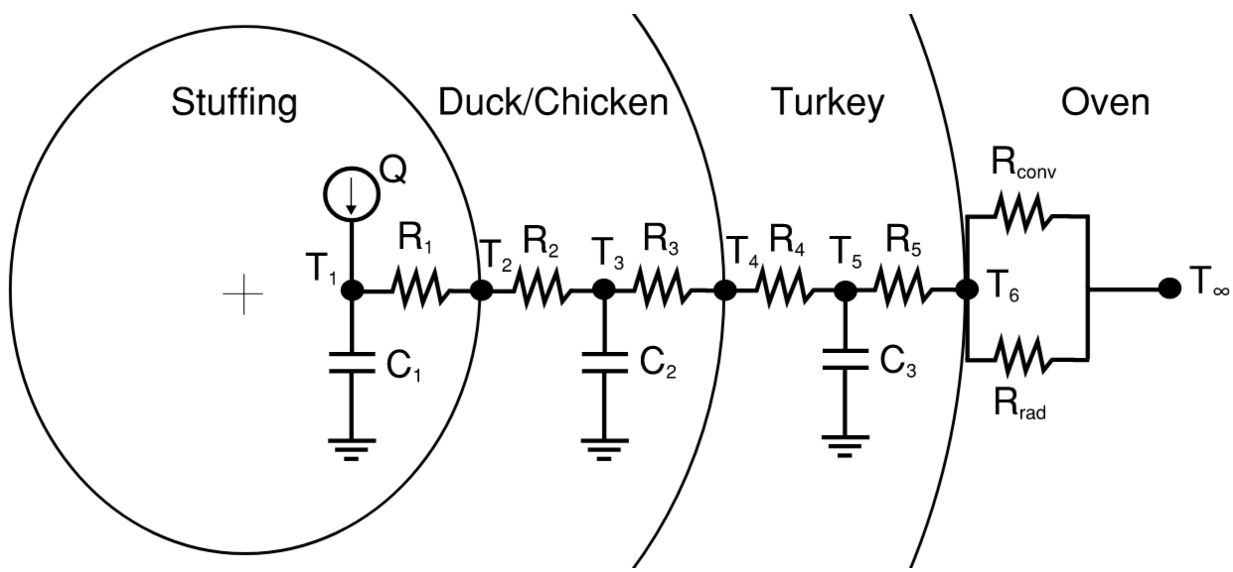

Figure 5: Equivalent circuit as described above

Now, the equivalent resistances can be split into two cases: an equivalent resistance based on thermal conductivity and an equivalent resistance based on convection and radiation. Resistances R_1 through R_5 are defined as equivalent resistances based on thermal conductivity, and the rightmost resistance is defined as an equivalent resistance based on the combination of convection and radiation.

The conductive resistance for a sphere is defined as:

$$R_k = \frac{r_2 - r_1}{4\pi k r_1 r_2},$$

where r_1 is the inside radius, r_2 is the outer radius, and k is the thermal conductivity of the sphere on which the node lies. This formulation is further explained in the MATLAB code in Appendix D. See Appendix B for the values of these resistances.

An equivalent resistance based on convection can be defined as:

$$R_h = \frac{1}{hA},$$

where h is the convection coefficient of the oven and A is the surface area of the outermost sphere. This formulation is further explained in the MATLAB code in Appendix D. See Appendix B for the values of this resistance.

A equivalent resistance based on radiation can be defined as:

$$R_r = \frac{1}{\varepsilon\sigma(T_s^{\,2} + T_\infty^{\,2})(T_s + T_\infty)A},$$

where ε is the emissivity of the outermost sphere surface and σ is the Stefan-Boltzmann constant. This formulation is further explained in the MATLAB code in Appendix D. See Appendix B for the values of this resistance.

An equivalent capacitance can be defined as:

$$C = \rho V c,$$

where$\rho$is the density of the sphere on which the node lies, V is the volume of the sphere on which the node lies, and c is the specific heat capacitance of the sphere on which the node lies. This formulation is further explained in the MATLAB code in Appendix D. See Appendix B for the values of these capacitances.

With all the numerical results tabulated, the circuit can now be solved. The system of coupled first order differential equations, with the current generator added to T_2, is listed below:

$$\frac{dT_1}{dt} = \left(\frac{1}{C_1}\right)\left(\frac{T_2 - T_1}{R_1} + Q_{dot}\right)$$

$$T_2 = \frac{\frac{T_1}{R_1} + \frac{T_3}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$\frac{dT_3}{dt} = \left(\frac{1}{C_3}\right)\left(\frac{T_2 - T_3}{R_2} + \frac{T_4 - T_3}{R_3}\right)$$

$$T_4 = \frac{\frac{T_3}{R_3} + \frac{T_5}{R_4}}{\frac{1}{R_3} + \frac{1}{R_4}}$$

$$\frac{dT_5}{dt} = \left(\frac{1}{C_5}\right)\left(\frac{T_4 - T_5}{R_4} + \frac{T_6 - T_5}{R_5}\right)$$

$$T_6 = \frac{\frac{T_5}{R_5} + \frac{T_\infty}{R_{eq}}}{\frac{1}{R_5} + \frac{1}{R_{eq}}}$$

$$R_{eq} = \frac{R_h R_r}{R_h + R_r},$$

where T_1, T_2, T_3, T_4, T_5, ,and T_6 are the nodal temperatures, $T_\infty = 350\ {}^oF$,and $Q_{dot} = 0.05213\ \frac{BTU}{s}$.

The initial conditions are given in the problem statement, as every temperature in the turducken is assumed to be 42 ºF at the beginning of the cooking process. In other words,

$$T_1(0) = 42\ {}^oF$$

$$T_2(0) = 42\ {}^oF$$

$$T_3(0) = 42\ {}^oF$$

$$T_4(0) = 42\ ^oF$$

$$T_5(0) = 42\ ^oF$$

$$T_6(0) = 42\ ^oF$$

Hand Calculations: Solution Method

As these differential equations are too difficult to solve analytically, numerical methods are used to solve this system. For the sake of accuracy, the Forward Euler method is employed. This method uses a finite change in time to approximate the solution at each time step.

This procedure is repeated for each of the nodes listed above, and temperature curves as a function of time are generated. It has been determined that the skewers should remain on until 4.25 hours (15,300 seconds) have passed. At this point, the skewers will be turned off. Below is a graph demonstrating the results until this point.

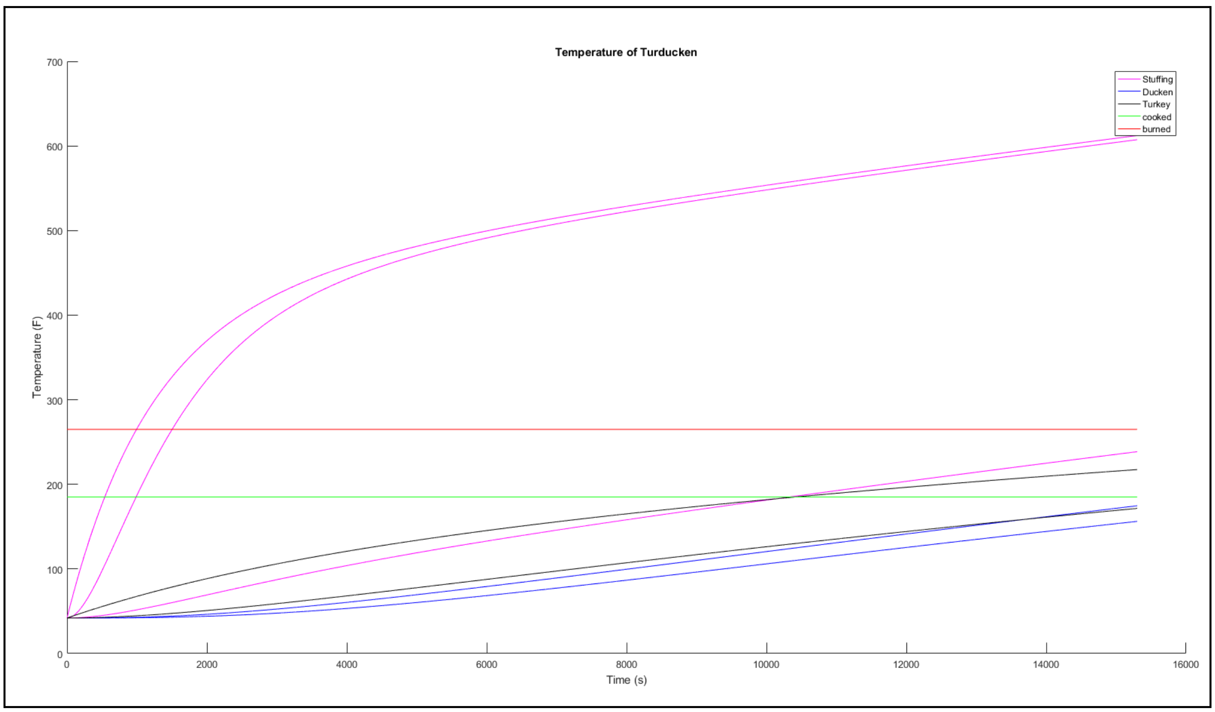

Figure 6: Graph of turducken temperature (°F) with respect to time (s) with $Q_{dot} = 0.05213\ \frac{BTU}{s}$ *for 15,300 seconds*

After this time, the skewers are turned off, and the governing differential equations are adjusted to make $Q_{dot} = 0$. In other words,

$$\frac{dT_1}{dt} = \left(\frac{1}{C_1}\right)\left(\frac{T_2 - T_1}{R_1}\right)$$

$$T_2 = \frac{\frac{T_1}{R_1} + \frac{T_3}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$\frac{dT_3}{dt} = \left(\frac{1}{C_3}\right)\left(\frac{T_2 - T_3}{R_2} + \frac{T_4 - T_3}{R_3}\right)$$

$$T_4 = \frac{\frac{T_3}{R_3} + \frac{T_5}{R_4}}{\frac{1}{R_3} + \frac{1}{R_4}}$$

$$\frac{dT_5}{dt} = \left(\frac{1}{C_5}\right)\left(\frac{T_4 - T_5}{R_4} + \frac{T_6 - T_5}{R_5}\right)$$

$$T_6 = \frac{\frac{T_5}{R_5} + \frac{T_\infty}{R_{eq}}}{\frac{1}{R_5} + \frac{1}{R_{eq}}}$$

$$R_{eq} = \frac{R_h R_r}{R_h + R_r},$$

where T_1, T_2, T_3, T_4, T_5, and T_6 are the nodal temperatures, and $T_\infty = 350\ ^oF$. The magnitude of T_∞ remains the same because the oven remains at the same temperature as the turkey cooks.

The initial conditions for this analysis are equivalent to the final temperatures at each node after the first analysis. It was found that the analysis ran best when the time spent in the oven is 6.25 hours (22,500 seconds). Using the Forward Euler approximation method as described above again, the following graph is obtained:

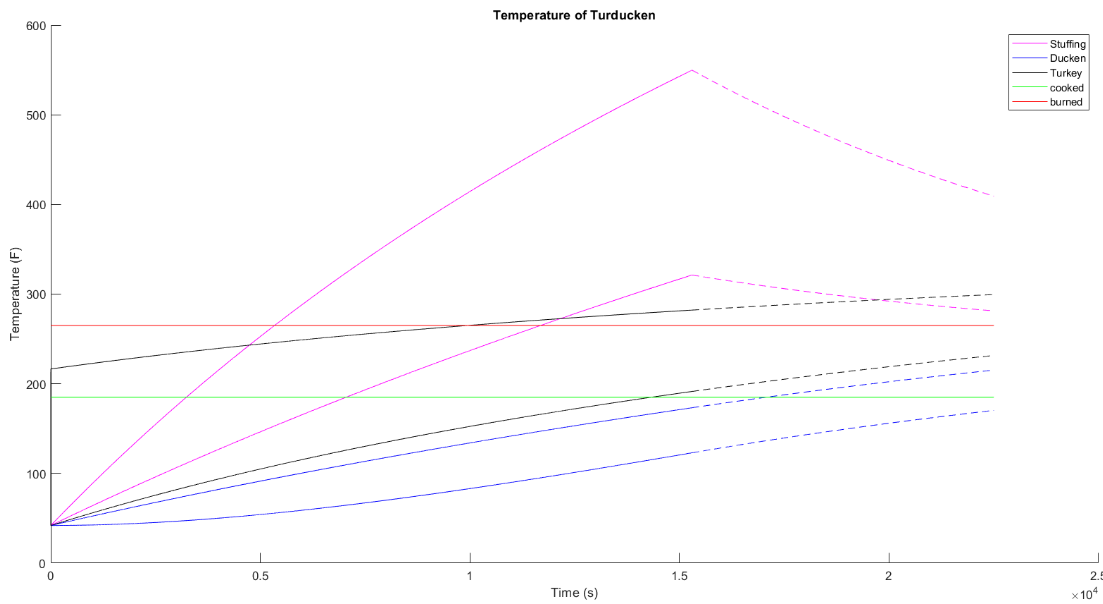

Figure 7: Graph of turducken temperature (ºF) with respect to time (s) for 22,500 seconds with $Q_{dot} = 0.05213 \, \frac{BTU}{s}$ *for the first 15,300 seconds*

Finally, if the full analysis is ran with the skewer turned off ($Q_{dot} = 0$) for the entire time period, the following curve is obtained:

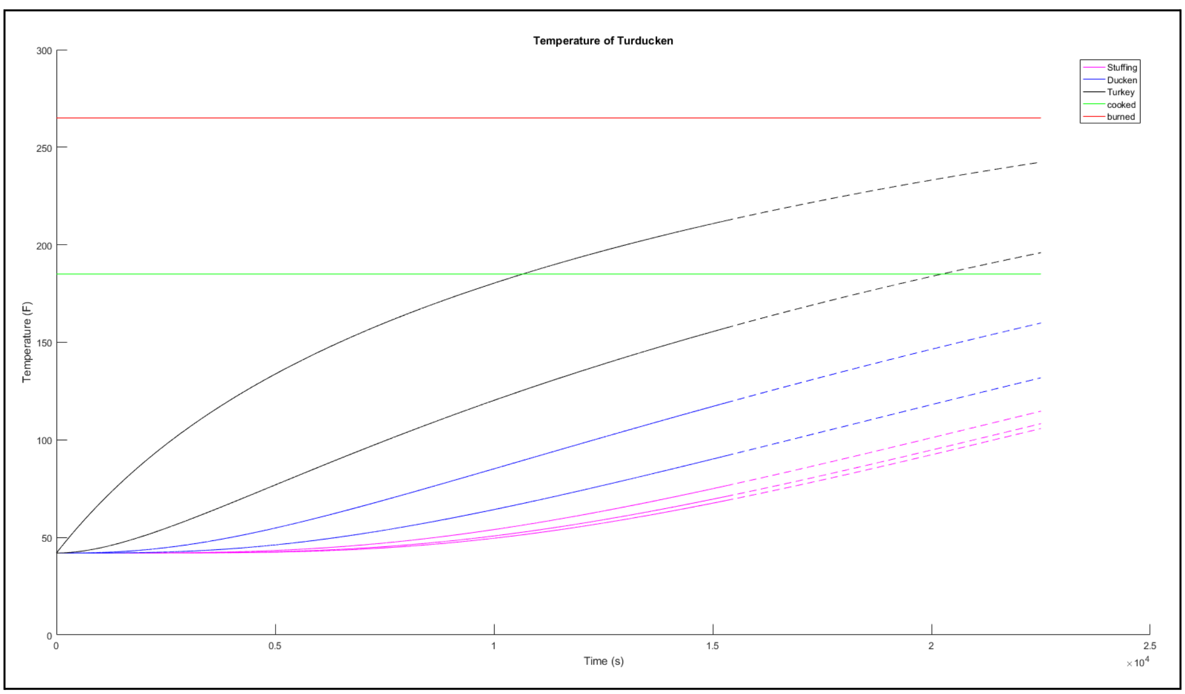

Figure 8: Graph of turducken temperature (ºF) with respect to time (s) for 22,500 seconds with $Q_{dot} = 0$ *for the entire time period*

From these figures, it can be seen that the skewers are necessary to heat the turducken to a cooked temperature in 6.25 hours. Thus, the skewers in their current position serve the intended purpose of both cooking a turducken in a more reasonable amount of time and minimizing potential burning. These hand calculation results help verify the simulation results, as shown in the Conclusion section.

Mesh

To reduce element count and computational time, the turducken is cut in half lengthwise and only half of the turducken is modeled. A symmetry region is applied in ANSYS to simulate the entire turducken. The stuffing, ducken, wing, and skewer are sliced to allow mapped meshing where possible, further reducing the element count. The stuffing, ducken, and wing are meshed with 0.25” elements. The skewer is meshed using 0.1” elements and is surrounded by a region of 0.1” tetrahedral elements. The turkey itself is meshed using a different approach, as it could not be easily sliced to allow for mapped meshing. A hexcore mesh is used instead, which creates a core of perfect cubes in the center of the turkey surrounded by a layer of tetrahedral elements. The final mesh has a total of 495,000 elements. The sizing of 0.25” allows for 4 elements across the ducken, which is the thinnest part of the turducken’s cross section, and the 0.1” tetrahedral elements allow for high resolution where there is a large temperature gradient (around the skewer). For comparison, a tetrahedral mesh with a similar resolution results in nearly one million elements. The sliced turducken geometry is shown below.

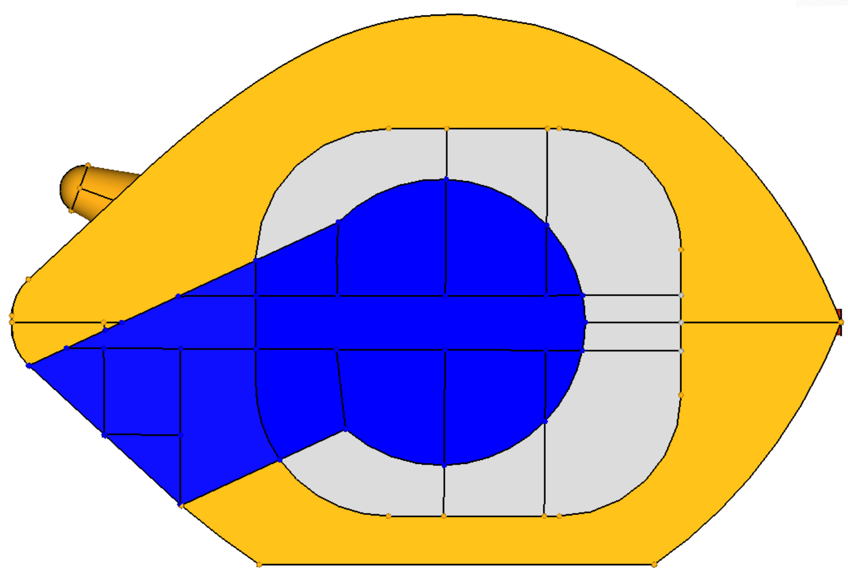

Figure 9: Sliced turducken geometry

The table below details which elements are used, along with their sizes and element counts. The figure just below the table designates each element section with a color. The blue, white, and orange elements are associated with the stuffing, ducken, and wing respectively. The red and purple elements represent the turkey, with the red being the hexcore and the purple being a tetrahedral volume mesh surrounding it.

Meshing Summary				
Section	**Color**	**Mesh Method**	**Sizing**	**Number of Elements**
Skewer	White	Mostly mapped	0.1”	13,000
Stuffing	Blue	Mostly mapped, except around skewer	0.1” around skewer, 0.25” elsewhere	90,000
Ducken	White	Mostly mapped, except around skewer	0.1” around skewer, 0.25” elsewhere	92,000
Turkey	Purple, Green, Red	CFD Hexcore	0.25”	281,000
Wing	Orange	Mostly mapped, except around boundary to turkey	0.25”	19,000
Total	~495,000 elements			

Table 1: Mesh sections, colors, mesh method, element sizes, and element counts

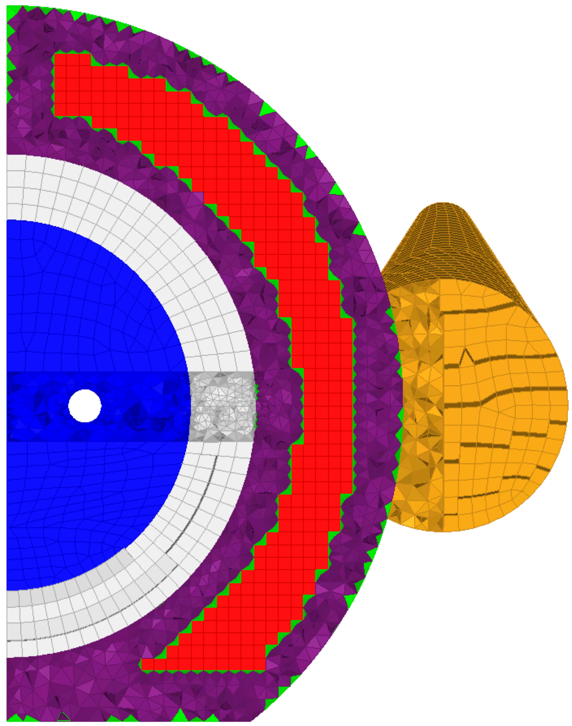

Figure 10: Cross-section of the final mesh. Colors on the mesh correspond with elements on the table

Boundary Conditions

All outer surfaces of the turducken have a convection and radiation boundary condition applied to them. The convection coefficient of a forced convection oven is 7.34×10^{-4} BTU/s ft^2 ^{o}F (2) (3). The emissivity of the turkey surface is 0.2 (2).

It is assumed that the bottom of the turducken rests on a thin aluminum pan, similar to the thin disposable aluminum pans found in most grocery stores. These pans are sufficiently thin (~0.1") such that conduction across them is negligible. Heat is transferred from the air in the oven to the pan by convection and then through the pan into the turkey by conduction. Since conduction is negligible (as previously stated), a convection boundary condition is applied to the bottom of the turkey with the convective heat transfer coefficient of aluminum (2.45×10^{-3} BTU/s ft^2 ^{o}F) (4) (5).

To model the resistive heating elements in one of the skewers, a heat flow boundary condition is applied to a portion of this skewer (see the Skewer Design section for a more detailed explanation). The magnitude of the heat flow is determined by experimentation and simulation (discussed in detail in the Skewer Design section), and the final value is determined to be 5.213×10^{-2} BTU/s. The figures outlining all the boundary conditions and a table summarizing these results are shown below.

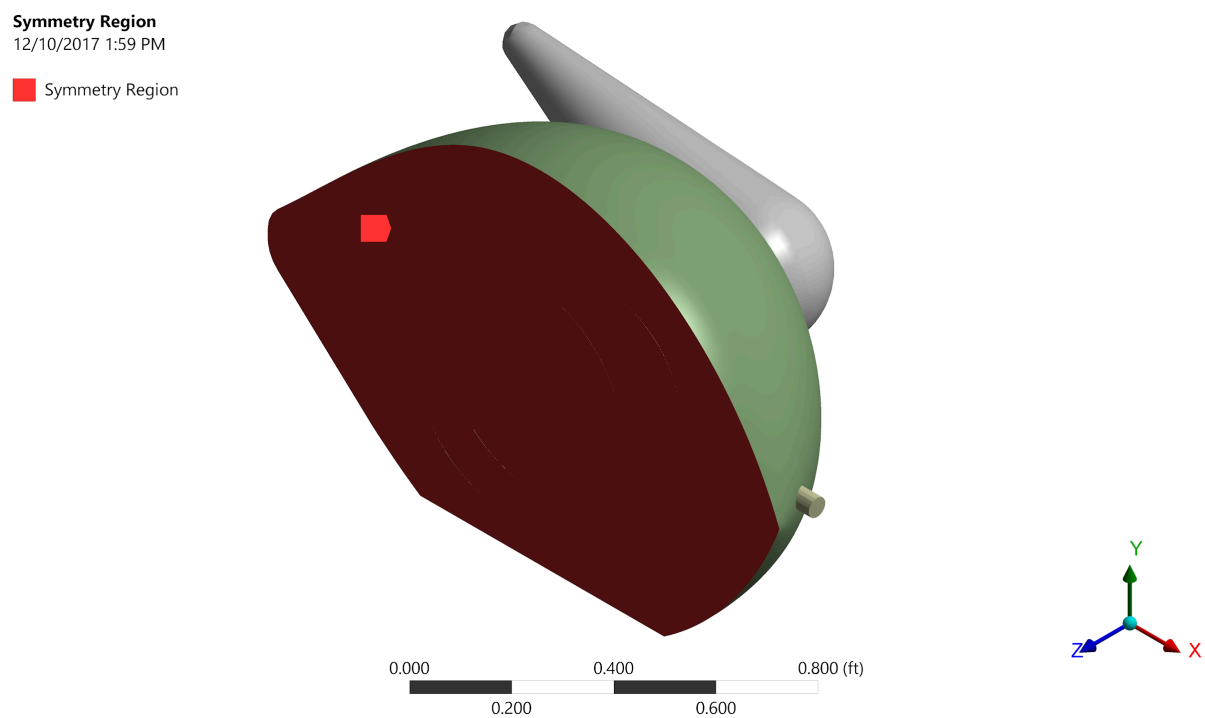

Figure 11: The symmetry condition applied to the plane about which ANSYS is to mirror to produce the full turducken

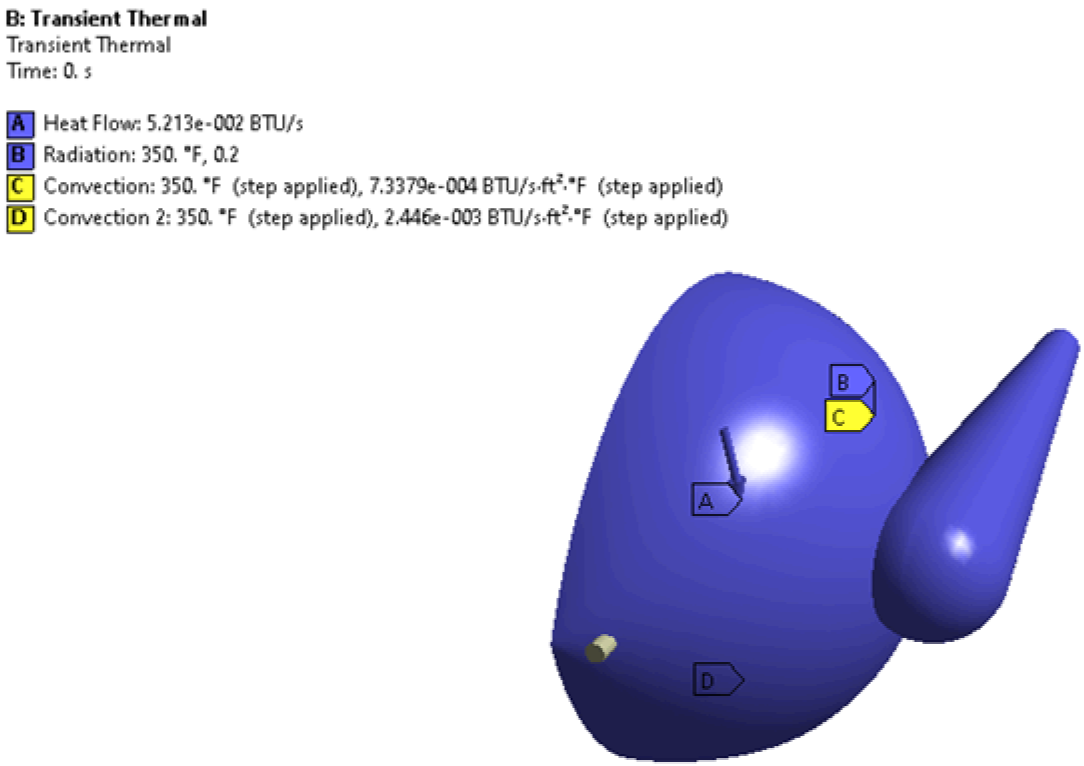

Figure 12: The thermal loads applied to the turducken

Boundary Conditions Summary		
Boundary Condition	**Location**	**Value**
Convection	Outer faces of turducken (excluding bottom face)	7.34×10^{-4} BTU/s ft^2 ^{o}F
Convection	Bottom face of turducken	2.45×10^{-3} BTU/s ft^2 ^{o}F
Heat flow	Skewer	5.213×10^{-2} BTU/s
Ambient temperature	Global	350 ^{o}F
Initial temperature	Global	42 ^{o}F

Table 2: Boundary conditions, their locations, and their numerical values

Initial Cooking Results (No Skewers)

An initial simulation is conducted to determine baseline cook times and to improve upon placement of the skewers. The simulation is run at a low temperature of 350 ºF for a long time interval to minimize the percentage of the turducken burned. The results of this initial simulation show a cook time of 12.3 hours (44,280 seconds), during which 60% of the turducken is burned (60% of elements reach a temperature of above 265 ºF). The results of the initial skewerless cook imply that heat sources symmetrically placed near the middle of the turducken would decrease cook time and may decrease burn percentage. Below are the temperature gradients ANSYS provides for the skewerless simulation from two different perspectives, along with a graph of minimum and maximum temperatures in the turducken as a function of time.

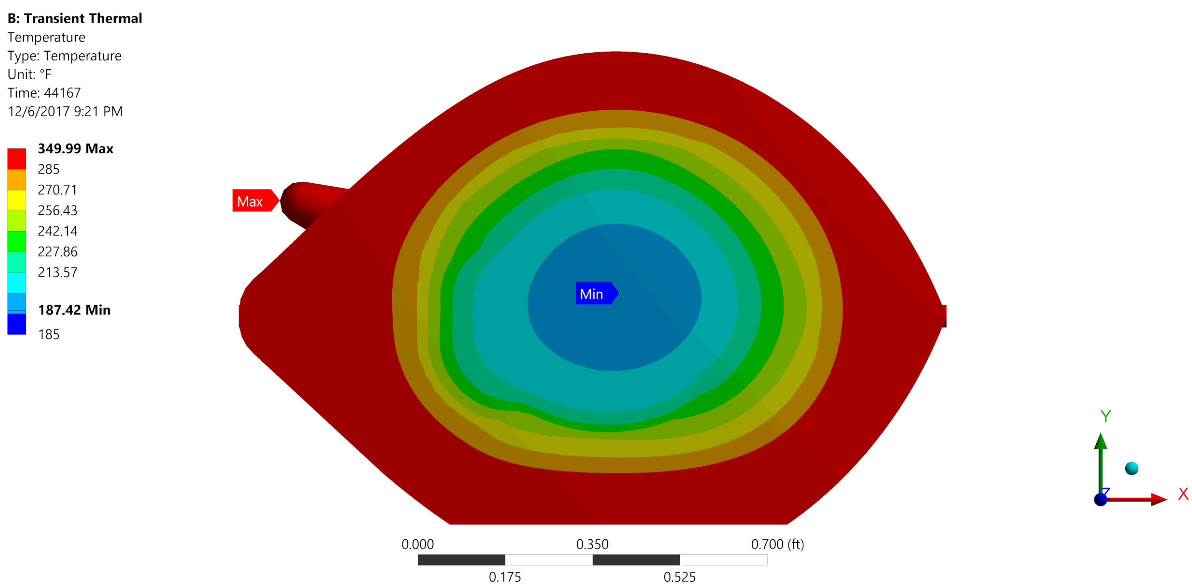

Figure 13: Skewerless temperature gradient (side view)

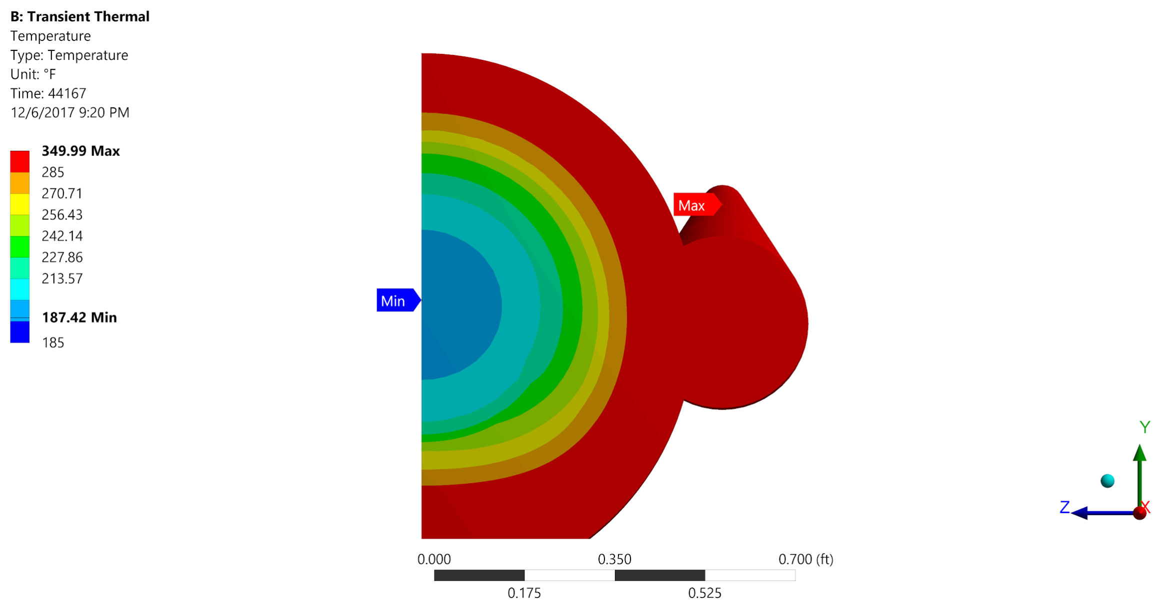

Figure 14: Skewerless temperature gradient (front view)

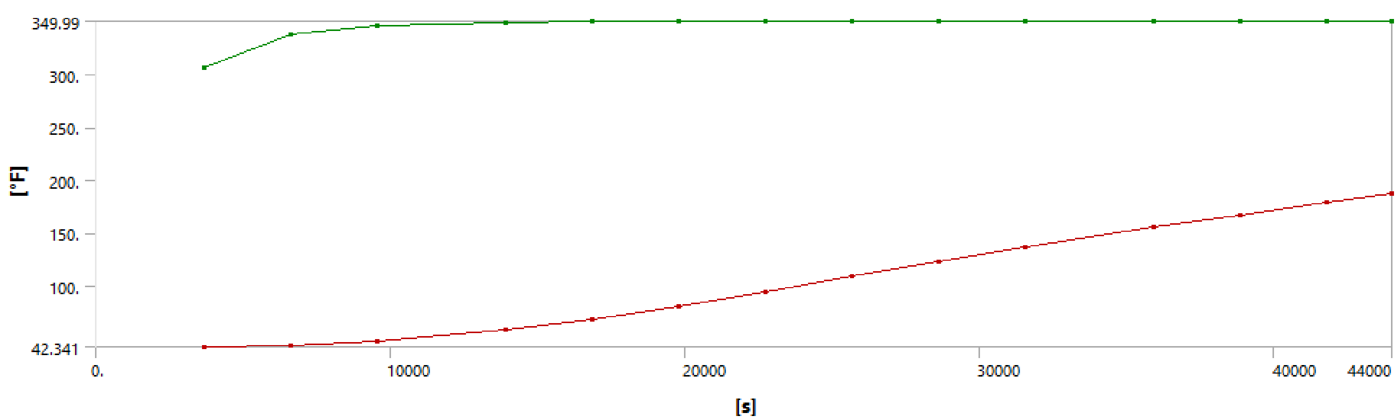

Figure 15: Graphical timeline:
Red: minimum temperature of the turducken
Green: maximum temperature of the turducken

Skewer Design

The skewer body is a cylinder for manufacturability and ease of insertion. There is a conical point at one end to aid in piercing. This conical point does not lie in the turducken when cooking.

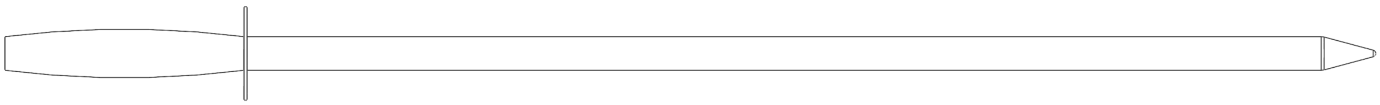

Figure 16: Proposed skewer design

Only the center part (highlighted in purple below) is heated.

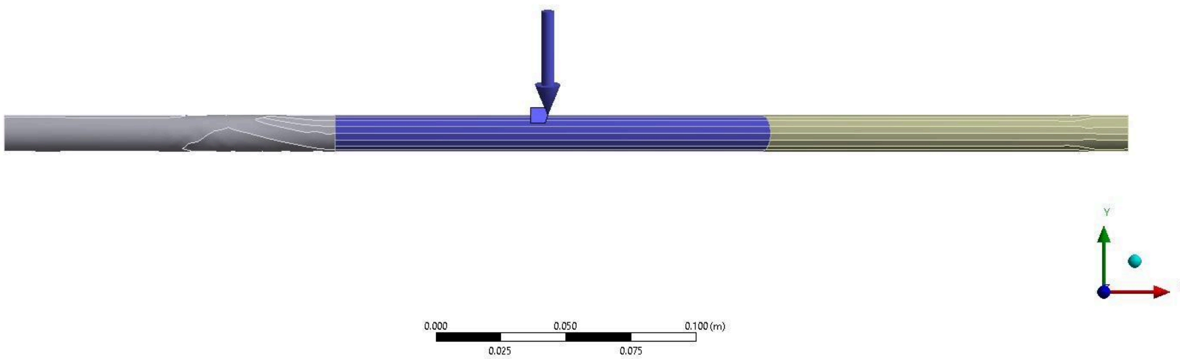

Figure 17: Proposed heated portion of skewer

The skewers are designed to be manufactured from molybdenum disilicide. It is a common heating element material used for high heat generation and is typically manufactured to sizes as used in the skewer (8). It is ceramic and has very well characterized thermal properties. It can be easily coated with a negligibly thin food-safe enamel with similar thermal properties to remove the possibility of ingestion (9).

The skewers are inserted symmetrically in the rear of the turducken at a point halfway up the body from the ground and about 1.25” both to the left and to the right of the center of the turducken. The skewers are inserted horizontally and pushed in up to the hilt. The skewer placement is motivated by the knowledge that all surrounding material will be irrevocably and very quickly burned. The skewer placement is chosen to maximize heating area while minimizing the contact area with the turducken itself. Below are diagrams from two different perspectives designating where the skewers should be placed.

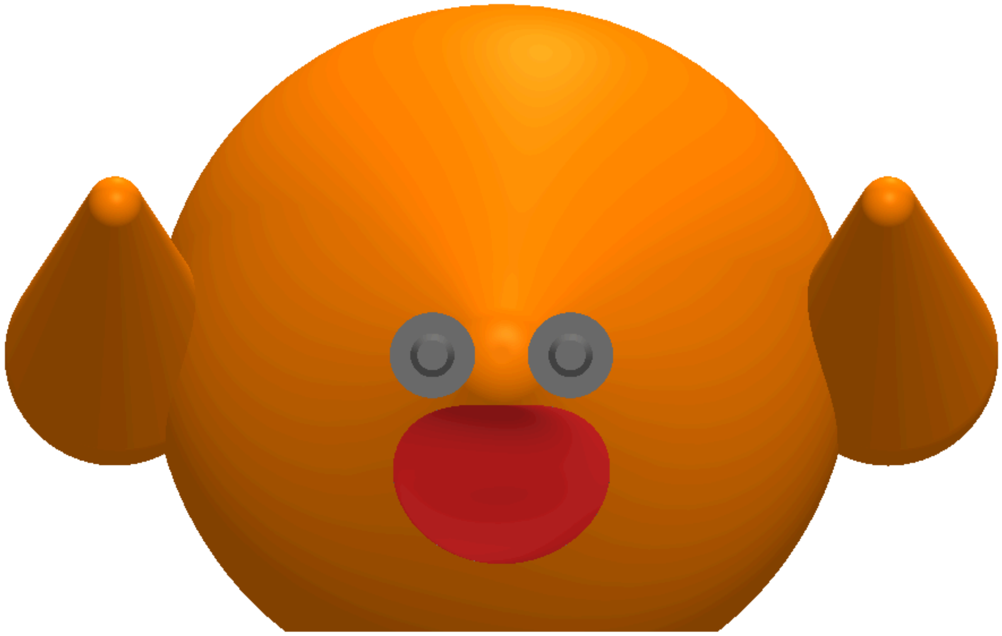

Figure 18: Proposed skewer placement (rear view)

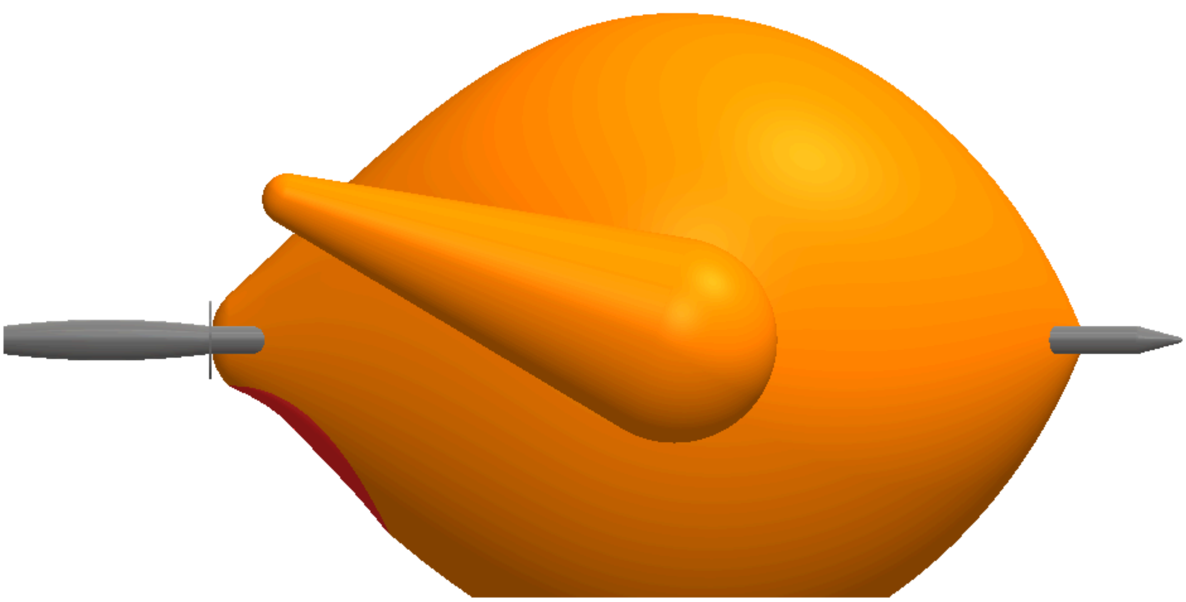

Figure 19: Proposed skewer placement (side view)

<u>Final Cooking Results</u>

After multiple iterations, the final cooking time for the turducken is chosen to be 6.25 hours, with two 0.05213 BTU/s skewers supplying extra heat to the turducken for 4.25 of those hours. The oven temperature is set at a constant 350ºF during this process. This cooking procedure is chosen to minimize the percentage of the turducken that is burned. Leaving the skewers on for 4.25 hours allows the center of the stuffing to heat up while minimizing the amount of burned food surrounding the skewers. The maximum temperature is close to 600ºF. However, this maximum is concentrated at the surface of the skewer. Further away in the ducken, the temperature is only 138ºF. Below are diagrams of a timeline designating the full cooking procedure and the temperature gradients of the turducken at the time the skewers are turned off from two different perspectives.

Hour	1	2	3	4	5	6	7
Turducken in Oven at 350 F							6.25 HOURS
Skewer on					4.25 HOURS		

Figure 20: Proposed timeline of entire cooking procedure

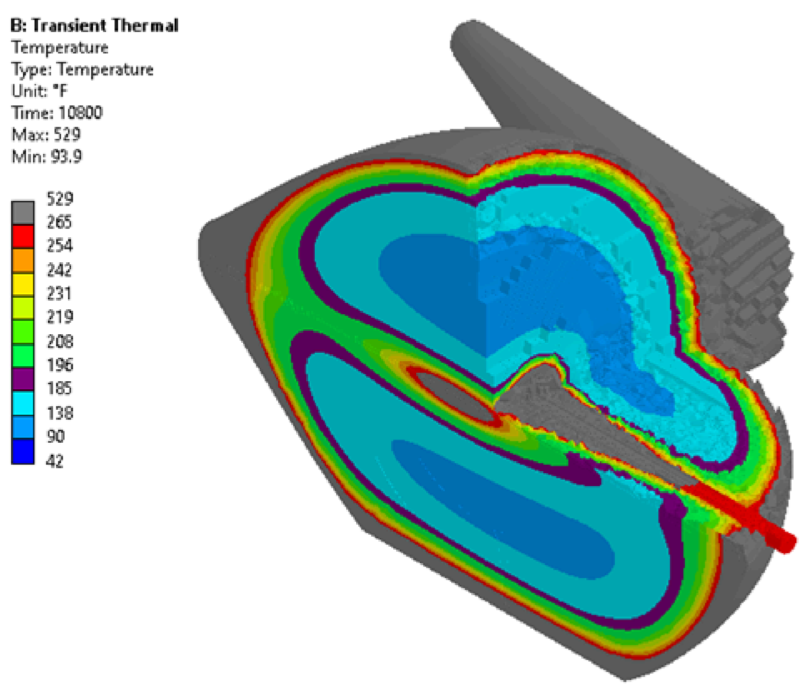

Figure 21: Skewered temperature gradient during hour 3 of cooking

After the skewers are turned off, the turducken is left in the oven for another 2 hours. This allows for the heat concentrated around the skewers to dissipate and cook the rest of the ducken. The maximum temperature is that of the oven (350ºF) on the surface of the wing. The minimum temperature in the turducken is just above the 185 ºF required to be fully cooked. Below are the temperature gradients of the turducken at the end of the cooking process.

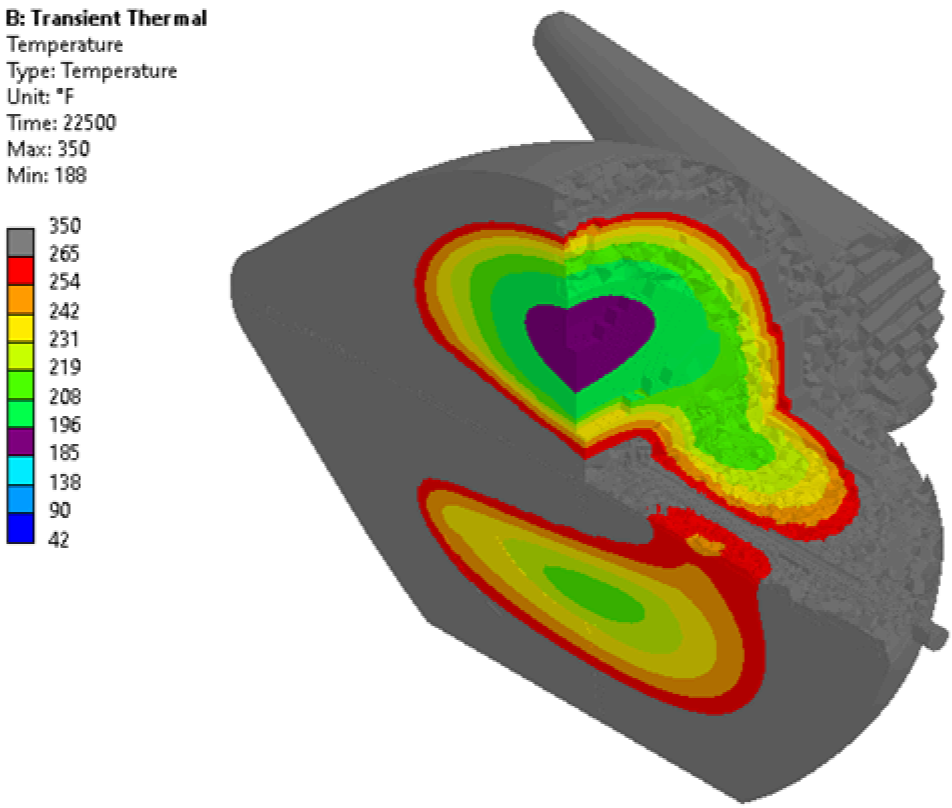

Figure 22: Skewered temperature gradient at the end of the cooking process

The maximum and minimum temperatures of the turducken over time are shown on the graph below. In order to reduce computational time and disk space, the results are stored at ~50 intervals.

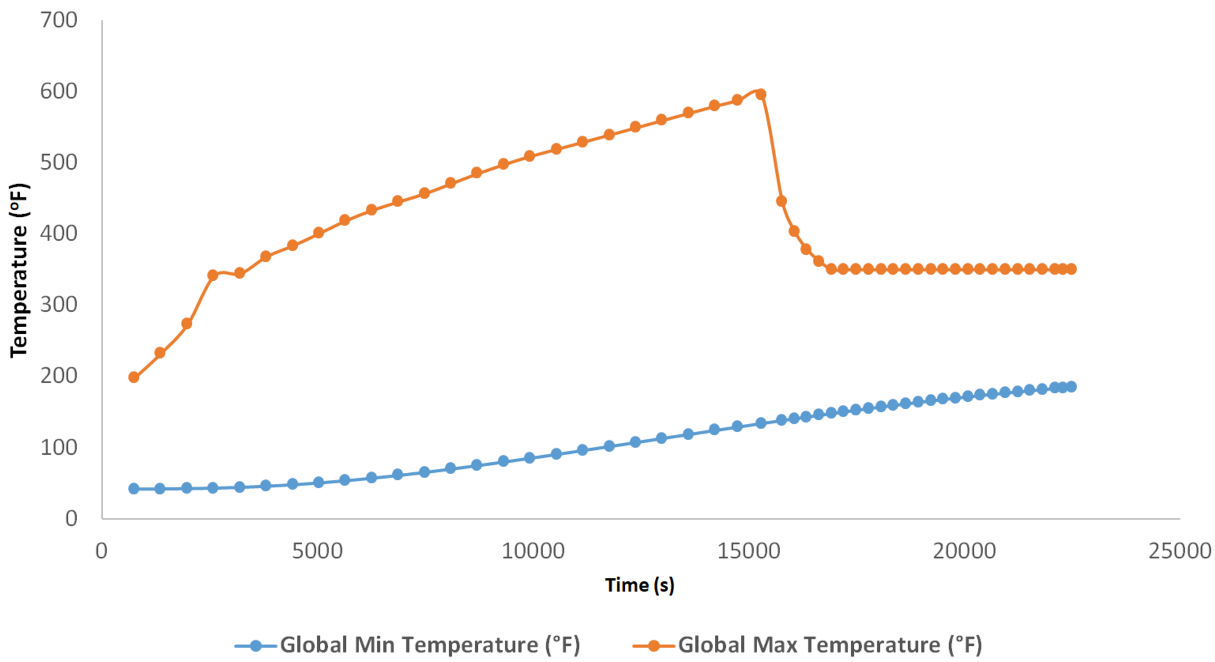

Figure 23: Graph of maximum temperature (ºF) and minimum temperature (ºF) with respect to time (s) over cooking procedure

Burned Percentages

A MATLAB script is used to determine the percentage of food burned, including the stuffing (see Appendix E for MATLAB code). The script inputs information about the model, such as nodal temperatures, element volumes, element densities, and a list of nodes for each element. Using this information, elements with a temperature above 265ºF are identified. To qualify as burned, the element's temperature has to exceed 265ºF when the skewers are turned off or when the turducken is taken out of the oven. Both times must be analyzed because some elements cool below 265ºF when the skewers are turned off. The formulas used to calculate the percentage of volume burned and mass burned are:

$$\%\ Volume\ Burned\ = \frac{Volume\ Burned}{Total\ volume\ of\ turducken}$$

$$\%\ Mass\ Burned\ = \frac{(Vol.\ Stuffing\ Burned)*(\rho_{stuffing}) + (Vol.\ Ducken\ Burned)*(\rho_{ducken}) + (Vol.\ Turkey\ Burned)*(\rho_{turkey})}{Total\ mass\ of\ turducken},$$

where $\rho_{stuffing}$, ρ_{ducken}, and ρ_{turkey} are the densities of stuffing, ducken, and turkey, respectively.

This script is used to compare burned percentages for skewered and skewerless analyses. The results that were calculated in the skewerless analysis show that the volume percentage burned is 60.4%, and the mass percentage burned is 61.0%. The skewered cooking process, however, results in a volume percentage burned of 39.0% and a mass percentage burned of 39.6%. Thus, with the skewers in the position indicated by the simulation and with the stuffing ingredients as described in the Stuffing Recipe section, the burned percentages are reduced by about 20% while reducing time in the oven by about 6 hours. Below is a figure of the results obtained from the MATLAB script in Appendix E and a table summarizing the numerical results discussed here.

```
Percent_burned_vol =

    0.3898


Percent_burned_mass =

    0.3957
```

Figure 24: MATLAB script results for burned volume and mass percentages for the skewered cooking process

Burned Percentages Summary		
	Skewerless	**Skewered**
Volume percentage burned	60.4%	39.0%
Mass percentage burned	61.0%	39.6%

Table 3: Burned percentage results

Conclusion

The turducken is around 40 pounds. The cooking method described in this paper yields about 24 pounds of edible, delicious meat. A standard Thanksgiving turkey is about 15-20 pounds (10) and a general rule of thumb is to cook the turkey for 18-20 minutes per pound (11). A 20 pound bird would require 360 minutes, or 6 hours. Our cooking method requires 6.25 hours for a turducken 200% of the size. As mentioned in the Burned Percentages section, when compared to a turducken of a similar size, the burned percentages are reduced by about 20% while reducing time in the oven by about 6 hours.

The turducken cooking method described in this paper yields a greater quantity of food than standard cooking time, or a faster cook time than the standard cooking protocol requires for the yield produced in this project. The figure below shows the simulation results superimposed on hand-calculated results.

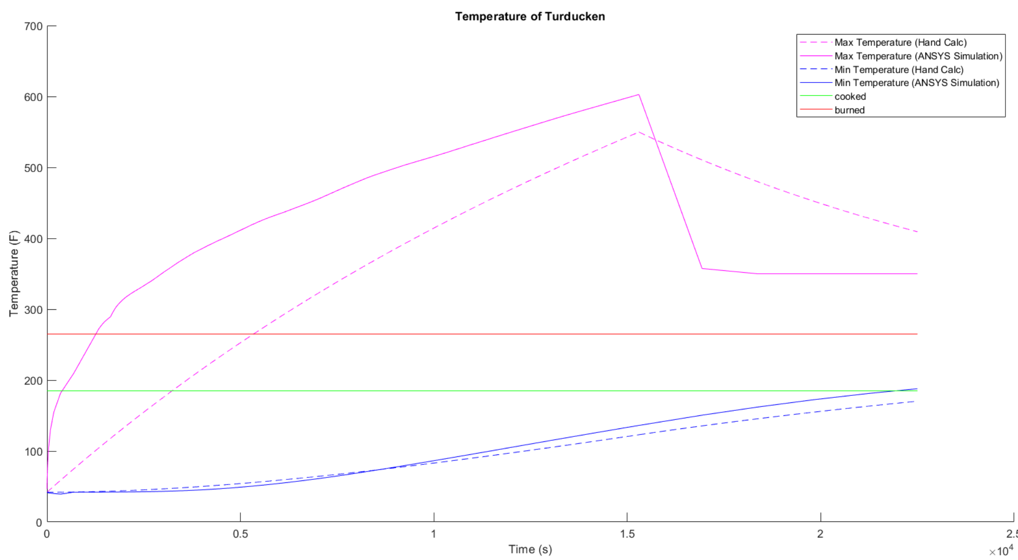

Figure 25: Simulation results superimposed on hand-calculated results

The approximate time that the team spent developing this design is approximately 150 hours.

Citations

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Appendix A: Thermal Property Calculations and Values

Summary: Thermal Constants Used

	Temperature (°F)	Turkey	Ducken	Stuffing
Specific heat (Btu/lb*F)	**42-212**	0.839	0.803	0.887
	212-265	0.466	0.683	0.273
	265+	0.257	0.257	0.257
Density (lb/ft^3)	**42-212**	65.8	60.3	65.8
	212-265	75.6	48.1	134
	265+	152	152	152
Conductivity (Btu/hr*ft*F)	**42-212**	0.232	0.188	0.266
	212-265	0.0976	0.173	0.0574
	265+	0.176	0.176	0.176

Detailed calculations and procedures are described below.

Initial Thermal Properties (42 °F - 212 °F):

Taken from ASHRAE (1). See "Thermal Properties" section for the underlying formulas.

Turkey						
	Moisture	Protein	Fat	Carb	Fiber	Ash
% Composition	70.4	20.42	8.02	0	0	0.88
Thermal Conductivity (Btu/hr*ft*F)	3.11E-01	9.05E-02	1.07E-01	1.01E-01	9.25E-02	1.76E-01
Density (lb/ft^3)	6.22E+01	8.36E+01	5.82E+01	1.00E+02	8.23E+01	1.52E+02
Specific Heat (Btu/lb*F)	9.98E-01	4.74E-01	4.67E-01	3.61E-01	4.33E-01	2.57E-01
Mass fraction (with water)	0.704	0.2042	0.0802	0	0	0.0088
Mass fraction / rho	1.13E-02	2.44E-03	1.38E-03	0	0	5.79E-05
Volume fraction	7.45E-01	1.61E-01	9.07E-02	0	0	3.81E-03
Volume fraction * k	2.32E-01	1.45E-02	9.70E-03	0	0	6.71E-04

Specific Heat * Mass fraction	7.03E-01	9.69E-02	3.75E-02	0	0	2.26E-03
Specific heat (Btu/lb*F)	**8.39E-01**					
Density (lb/ft^3)	**6.58E+01**					
Conductivity (Btu/hr*ft*F)	**2.32E-01**					

Duck						
	Moisture	Protein	Fat	Carb	Fiber	Ash
% Composition	48.5	11.49	39.34	0	0	0.68
Thermal Conductivity (Btu/hr*ft*F)	3.11E-01	9.05E-02	1.07E-01	1.01E-01	9.25E-02	1.76E-01
Density (lb/ft^3)	6.22E+01	8.36E+01	5.82E+01	1.00E+02	8.23E+01	1.52E+02
Specific Heat (Btu/lb*F)	9.98E-01	4.74E-01	4.67E-01	3.61E-01	4.33E-01	2.57E-01
Mass fraction (with water)	0.485	0.1149	0.3934	0	0	0.0068
Mass fraction / rho	7.80E-03	1.37E-03	6.76E-03	0	0	4.47E-05
Volume fraction	5.13E-01	9.04E-02	4.45E-01	0	0	2.94E-03
Volume fraction * k	1.60E-01	8.18E-03	4.76E-02	0	0	5.18E-04
Specific Heat * Mass fraction	4.84E-01	5.45E-02	1.84E-01	0	0	1.74E-03
Specific heat (Btu/lb*F)	**7.24E-01**					
Density (lb/ft^3)	**6.26E+01**					
Conductivity (Btu/hr*ft*F)	**1.60E-01**					

Chicken						
	Moisture	Protein	Fat	Carb	Fiber	Ash
% Composition	65.99	18.6	15.06	12.68	4.2	0.19
Thermal Conductivity (Btu/hr*ft*F)	3.11E-01	9.05E-02	1.07E-01	1.01E-01	9.25E-02	1.76E-01
Density (lb/ft^3)	6.22E+01	8.36E+01	5.82E+01	1.00E+02	8.23E+01	1.52E+02
Specific Heat (Btu/lb*F)	9.98E-01	4.74E-01	4.67E-01	3.61E-01	4.33E-01	2.57E-01
Mass fraction (with water)	0.6599	0.186	0.1506	0.1268	0.042	0.0019
Mass fraction / rho	1.06E-02	2.22E-03	2.59E-03	1.27E-03	5.10E-04	1.25E-05
Volume fraction	6.98E-01	1.46E-01	1.70E-01	8.34E-02	3.36E-02	8.23E-04

Volume fraction * k	2.17E-01	1.32E-02	1.82E-02	8.45E-03	3.11E-03	1.45E-04
Specific Heat * Mass fraction	6.59E-01	8.82E-02	7.04E-02	4.58E-02	1.82E-02	4.88E-04
Specific heat (Btu/lb*F)	**8.82E-01**					
Density (lb/ft^3)	**5.81E+01**					
Conductivity (Btu/hr*ft*F)	**2.17E-01**					

Ducken						
Specific heat (Btu/lb*F)	**8.03E-01**					
Density (lb/ft^3)	**6.03E+01**					
Conductivity (Btu/hr*ft*F)	**1.88E-01**					

Potatoes						
	Moisture	Protein	Fat	Carb	Fiber	Ash
% Composition	78.96	2.07	0.1	17.98	1.6	0.89
Thermal Conductivity (Btu/hr*ft*F)	3.11E-01	9.05E-02	1.07E-01	1.01E-01	9.25E-02	1.76E-01
Density (lb/ft^3)	6.22E+01	8.36E+01	5.82E+01	1.00E+02	8.23E+01	1.52E+02
Specific Heat (Btu/lb*F)	9.98E-01	4.74E-01	4.67E-01	3.61E-01	4.33E-01	2.57E-01
Mass fraction (with water)	0.7896	0.0207	0.001	0.1798	0.016	0.0089
Mass fraction / rho	1.27E-02	2.48E-04	1.72E-05	1.80E-03	1.94E-04	5.86E-05
Volume fraction	8.35E-01	1.63E-02	1.13E-03	1.18E-01	1.28E-02	3.85E-03
Volume fraction * k	2.60E-01	1.47E-03	1.21E-04	1.20E-02	1.18E-03	6.78E-04
Specific Heat * Mass fraction	7.88E-01	9.82E-03	4.67E-04	6.49E-02	6.92E-03	2.28E-03
Specific heat (Btu/lb*F)	**8.73E-01**					
Density (lb/ft^3)	**6.66E+01**					
Conductivity (Btu/hr*ft*F)	**2.60E-01**					

Cranberries						
	Moisture	Protein	Fat	Carb	Fiber	Ash

% Composition	86.54	0.39	0.2	12.68	4.2	0.19
Thermal Conductivity (Btu/hr*ft*F)	3.11E-01	9.05E-02	1.07E-01	1.01E-01	9.25E-02	1.76E-01
Density (lb/ft^3)	6.22E+01	8.36E+01	5.82E+01	1.00E+02	8.23E+01	1.52E+02
Specific Heat (Btu/lb*F)	9.98E-01	4.74E-01	4.67E-01	3.61E-01	4.33E-01	2.57E-01
Mass fraction (with water)	0.8654	0.0039	0.002	0.1268	0.042	0.0019
Mass fraction / rho	1.39E-02	4.67E-05	3.44E-05	1.27E-03	5.10E-04	1.25E-05
Volume fraction	9.16E-01	3.07E-03	2.26E-03	8.34E-02	3.36E-02	8.23E-04
Volume fraction * k	2.85E-01	2.78E-04	2.42E-04	8.45E-03	3.11E-03	1.45E-04
Specific Heat * Mass fraction	8.64E-01	1.85E-03	9.35E-04	4.58E-02	1.82E-02	4.88E-04
Specific heat (Btu/lb*F)	**9.31E-01**					
Density (lb/ft^3)	**6.34E+01**					
Conductivity (Btu/hr*ft*F)	**2.85E-01**					

Stuffing						
% Potatoes	75					
% Cranberries	25					
Specific heat (Btu/lb*F)	**8.87E-01**					
Density (lb/ft^3)	**6.58E+01**					
Conductivity (Btu/hr*ft*F)	**2.66E-01**					

Thermal Properties without water (212 ºF - 265 ºF):

As explained in the “Thermal Properties” section, once an area of the turducken reaches 212 ºF it is assumed that all the moisture in that area boils away. In other words, the mass percentage of moisture becomes zero. The thermal properties of the ingredient after this temperature are calculated by first setting the mass percent of moisture to zero. Then, the mass percentages of the other constituents can be calculated and the thermal properties are determined.

Turkey						
	Moisture	Protein	Fat	Carb	Fiber	Ash
% Composition	70.4	20.42	8.02	0	0	0.88
Thermal Conductivity (Btu/hr*ft*F)	3.11E-01	9.05E-02	1.07E-01	1.01E-01	9.25E-02	1.76E-01

Density (lb/ft^3)	6.22E+01	8.36E+01	5.82E+01	1.00E+02	8.23E+01	1.52E+02
Specific Heat (Btu/lb*F)		4.74E-01	4.67E-01	3.61E-01	4.33E-01	2.57E-01
Mass fraction (with water)	0.704	0.2042	0.0802	0	0	0.0088
Mass fraction (no water)	-	0.696	0.274	0	0	0.030
Mass fraction / rho	-	8.33E-03	4.70E-03	0	0	1.97E-04
Volume fraction	-	6.30E-01	3.55E-01	0	0	1.49E-02
Volume fraction * k	-	5.70E-02	3.80E-02	0	0	2.63E-03
Specific Heat * Mass fraction		3.30E-01	1.28E-01	0	0	7.70E-03
Specific heat no water (Btu/lb*F)	**4.66E-01**					
Density no water (lb/ft^3)	**7.56E+01**					
Conductivity no water (Btu/hr*ft*F)	**9.76E-02**					

Duck						
	Moisture	Protein	Fat	Carb	Fiber	Ash
% Composition	48.5	11.49	39.34	0	0	0.68
Thermal Conductivity (Btu/hr*ft*F)	3.11E-01	9.05E-02	1.07E-01	1.01E-01	9.25E-02	1.76E-01
Density (lb/ft^3)	6.22E+01	8.36E+01	5.82E+01	1.00E+02	8.23E+01	1.52E+02
Specific Heat (Btu/lb*F)		4.74E-01	4.67E-01	3.61E-01	4.33E-01	2.57E-01
Mass fraction (with water)	0.485	0.1149	0.3934	0	0	0.0068
Mass fraction (no water)	-	0.392	1.342	0	0	0.023
Mass fraction / rho	-	4.69E-03	2.31E-02	0	0	1.53E-04
Volume fraction	-	3.54E-01	1.74E+00	0	0	1.15E-02
Volume fraction * k	-	3.21E-02	1.86E-01	0	0	2.03E-03
Specific Heat * Mass fraction		1.86E-01	6.27E-01	0	0	5.95E-03
Specific heat no water (Btu/lb*F)	**8.19E-01**					
Density no water (lb/ft^3)	**3.58E+01**					

Conductivity no water (Btu/hr*ft*F)	**2.21E-01**					

Chicken						
	Moisture	Protein	Fat	Carb	Fiber	Ash
% Composition	65.99	18.6	15.06	0	0	0.79
Thermal Conductivity (Btu/hr*ft*F)	3.11E-01	9.05E-02	1.07E-01	1.01E-01	9.25E-02	1.76E-01
Density (lb/ft^3)	6.22E+01	8.36E+01	5.82E+01	1.00E+02	8.23E+01	1.52E+02
Specific Heat (Btu/lb*F)		4.74E-01	4.67E-01	3.61E-01	4.33E-01	2.57E-01
Mass fraction (with water)	0.6599	0.186	0.1506	0	0	0.0079
Mass fraction (no water)	-	0.634	0.514	0	0	0.027
Mass fraction / rho	-	7.59E-03	8.83E-03	0	0	1.77E-04
Volume fraction	-	5.74E-01	6.67E-01	0	0	1.34E-02
Volume fraction * k	-	5.19E-02	7.14E-02	0	0	2.36E-03
Specific Heat * Mass fraction		3.01E-01	2.40E-01	0	0	6.91E-03
Specific heat no water (Btu/lb*F)	**5.48E-01**					
Density no water (lb/ft^3)	**6.03E+01**					
Conductivity no water (Btu/hr*ft*F)	**1.26E-01**					

Ducken						
Specific heat no water (Btu/lb*F)	**6.83E-01**					
Density no water (lb/ft^3)	**4.81E+01**					
Conductivity no water (Btu/hr*ft*F)	**1.73E-01**					

Potatoes						
	Moisture	Protein	Fat	Carb	Fiber	Ash
% Composition	78.96	2.07	0.1	17.98	1.6	0.89

Thermal Conductivity (Btu/hr*ft*F)	3.11E-01	9.05E-02	1.07E-01	1.01E-01	9.25E-02	1.76E-01
Density (lb/ft^3)	6.22E+01	8.36E+01	5.82E+01	1.00E+02	8.23E+01	1.52E+02
Specific Heat (Btu/lb*F)		4.74E-01	4.67E-01	3.61E-01	4.33E-01	2.57E-01
Mass fraction (with water)	0.7896	0.0207	0.001	0.1798	0.016	0.0089
Mass fraction (no water)	-	0.071	0.003	0.613	0.055	0.030
Mass fraction / rho	-	8.45E-04	5.86E-05	6.13E-03	6.63E-04	2.00E-04
Volume fraction	-	6.38E-02	4.43E-03	4.64E-01	5.01E-02	1.51E-02
Volume fraction * k	-	5.78E-03	4.74E-04	4.70E-02	4.64E-03	2.66E-03
Specific Heat * Mass fraction		3.35E-02	1.59E-03	2.21E-01	2.36E-02	7.79E-03
Specific heat no water (Btu/lb*F)	**2.88E-01**					
Density no water (lb/ft^3)	**1.27E+02**					
Conductivity no water (Btu/hr*ft*F)	**6.05E-02**					

Cranberries						
	Moisture	Protein	Fat	Carb	Fiber	Ash
% Composition	86.54	0.39	0.2	12.68	4.2	0.19
Thermal Conductivity (Btu/hr*ft*F)	3.11E-01	9.05E-02	1.07E-01	1.01E-01	9.25E-02	1.76E-01
Density (lb/ft^3)	6.22E+01	8.36E+01	5.82E+01	1.00E+02	8.23E+01	1.52E+02
Specific Heat (Btu/lb*F)		4.74E-01	4.67E-01	3.61E-01	4.33E-01	2.57E-01
Mass fraction (with water)	0.8654	0.0039	0.002	0.1268	0.042	0.0019
Mass fraction (no water)	-	0.013	0.007	0.432	0.143	0.006
Mass fraction / rho	-	1.59E-04	1.17E-04	4.32E-03	1.74E-03	4.26E-05
Volume fraction	-	1.20E-02	8.86E-03	3.27E-01	1.32E-01	3.22E-03
Volume fraction * k	-	1.09E-03	9.48E-04	3.31E-02	1.22E-02	5.67E-04
Specific Heat * Mass fraction		6.31E-03	3.19E-03	1.56E-01	6.20E-02	1.66E-03
Specific heat no water (Btu/lb*F)	**2.29E-01**					

Density no water (lb/ft^3)	**1.57E+02**					
Conductivity no water (Btu/hr*ft*F)	**4.79E-02**					
Stuffing						
% Potatoes	0.75					
% Cranberries	0.25					
Specific heat no water (Btu/lb*F)	**2.73E-01**					
Density no water (lb/ft^3)	**1.34E+02**					
Conductivity no water (Btu/hr*ft*F)	**5.74E-02**					

Thermal Properties of Ash (265 ºF +):

Ash	
Specific heat (Btu/lb*F)	2.57E-01
Density (lb/ft^3)	1.52E+02
Conductivity (Btu/hr*ft*F)	1.76E-01

Appendix B: Numerical Constants for Hand Calculations

Radial Distance Between Nodes (ft)	
r_1	0.26
r_2	0.49
r_3	0.69

Volumes $\left(ft^3\right)$	
V_1	0.0461
V_2	0.243
V_3	1.076

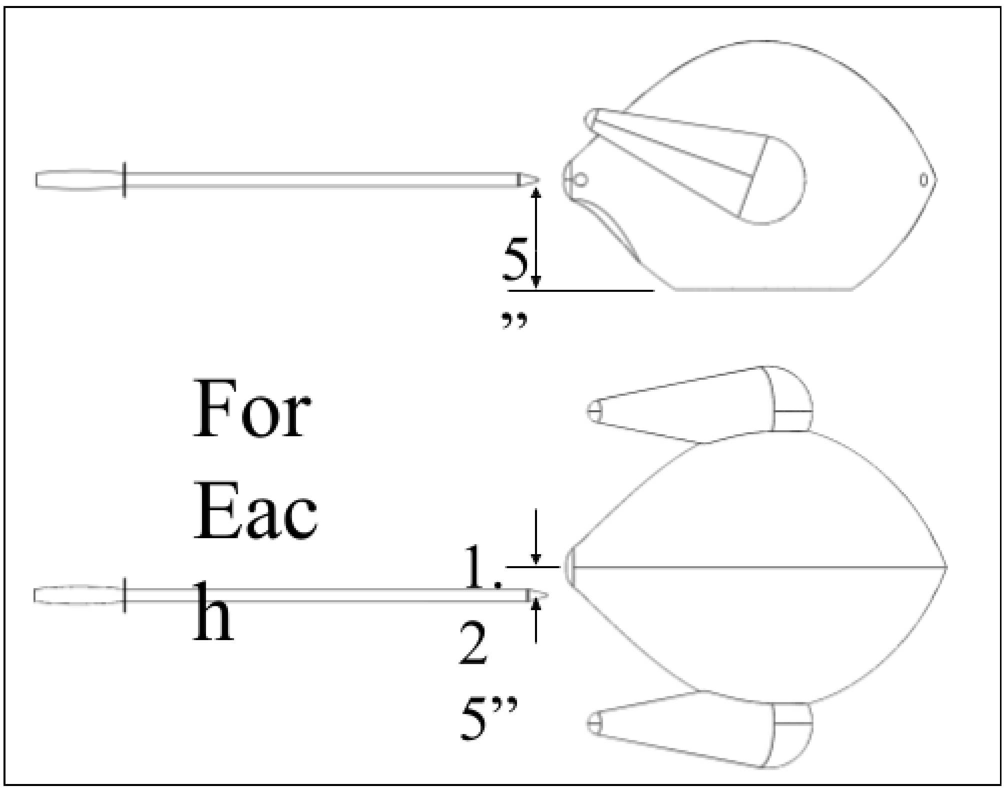
Resistances $\left(\frac{s\,^oF}{BTU}\right)$	
R_1	$1.59 \bullet 10^4$
R_2	$4.22 \bullet 10^3$
R_3	$1.76 \bullet 10^3$
R_4	$1.03 \bullet 10^3$
R_5	602

Capacitances $\left(BTU/^{o}F\right)$	
C_1	2.693
C_2	11.786
C_3	59.468

Miscellaneous Constants Used	
Heat output of skewer (Q_{dot} ($\frac{BTU}{s}$))	$5.213 \bullet 10^{-2}$
Convection coefficient of oven ($h1$ ($BTU/s\,ft^{2}\,^{o}F$))	$7.34 \bullet 10^{-4}$
Convection coefficient of pan ($h2$ ($BTU/s\,ft^{2}\,^{o}F$))	$2.45 \bullet 10^{-3}$
Emissivity of turkey surface	0.2
Temperature of oven (T_{∞} (^{o}F))	350

Appendix C: Cooking Instructions

1. Preheat oven to 350 degrees Fahrenheit.
2. Take turducken out of fridge and stuff with our delicious recipe.
3. Insert skewers as shown in the diagram.
4. Place turducken in oven on a thin disposable aluminum pan and turn on the skewers.
5. Set a timer for 4.25 hours, wait, relax, treat yourself.
6. When the timer goes off, turn off the skewers.
7. Cook the turducken for an additional 2 hours, resulting in 6.25 hours total cook time.
8. After 2 hours, remove your perfectly cooked turducken.
9. Enjoy!



Hour	1	2	3	4	5	6	7
Turducken in Oven at 350 F							6.25 HOURS
Skewer on					4.25 HOURS		

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Appendix D: Hand Calculations MATLAB Script

Turducken hand calculations using Forward Euler numerical methods

```
close all; clear; clc;
```

Initialized Geometry

The model is broken into three bodies, modeled as concentric spheres. The inner most body is the stuffing. The body beyond that is the ducken. The outermost body is the turkey. This is a total of three spheres. The spheres are 1D elements with only one node that is the entire surface of the sphere. It is assumed that each sphere has a uniform temperature equivalent to the surface temperature of that sphere.

The radii are of spheres with approxiimate volumes to the body they are elements of.

Average radii of stuffing, ducken, and turkey

```
r_stuffing = .26;   %[ft^2]
r_ducken = .49;     %[ft^2]
r_turkey = .69;     %[ft^2]
```

Surface area of outermost sphere

```
As = (4*pi*(r_turkey)^2);  %[ft^2]
```

Volume of spheres, starting from absolute center and moving radially outward

```
V_stuffing = (4/3)*pi*(r_stuffing)^3;                %[ft^3]
V_ducken = (4/3)*pi*(r_ducken^3 - r_stuffing^3);     %[ft^3]
V_turkey = (4/3)*pi*(r_turkey^3 - r_ducken^3);       %[ft^3]
```

Material Constants

The following thermal properties are held constant. Our ANSYS simulations are not constant due mainly to the evaporation of water. This preliminary calculation is to get us to the correct order of magnitude and should not need the complexity of changing material properties. The thermal properties of ducken are calculated to be an average of those of duck and chicken

Thermal conductivity of stuffing, ducken, and turkey, respectively:

```
k_stuffing = 0.266/(3600); %[BTU/ft*s*F]
```

```
k_ducken = 0.188/(3600); %[BTU/ft*s*F]
k_turkey = 0.287/(3600); %[BTU/ft*s*F]
```

Density of stuffing, ducken, and turkey, respectively:

```
rho_stuffing = 65.8; %[lb/ft^3]
rho_ducken = 60.3; %[lb/ft^3]
rho_turkey = 65.8; %[lb/ft^3]
```

Heat capacity of stuffing, ducken, and turkey, respectively:

```
c_stuffing = 0.887; %[BTU/lb*F]
c_ducken = 0.803; %[BTU/lb*F]
c_turkey = 0.84;  %[BTU/lb*F]
```

Convection coefficient of our chosen, conventional kitchen oven

```
h = 7.34e-4; %[BTU/s*ft^2*F]
```

Equivalent thermal resistance (using thermal conductivity).

```
R1 = (r_stuffing - (2*r_stuffing/3))/
(4*pi*r_stuffing*(2*r_stuffing/3)*k_stuffing);       %[s*F/BTU]
R2 = (((r_ducken+r_stuffing)/2) - r_stuffing)/(4*pi*((r_ducken
+r_stuffing)/2)*r_stuffing*k_ducken);                 %[s*F/BTU]
R3 = (r_ducken - ((r_ducken+r_stuffing)/2))/(4*pi*r_ducken*((r_ducken
+r_stuffing)/2)*k_ducken);                      %[s*F/BTU]
R4 = (((r_turkey+r_ducken)/2) - r_ducken)/(4*pi*((r_turkey
+r_ducken)/2)*r_ducken*k_turkey);                     %[s*F/BTU]
R5 = (r_turkey - ((r_turkey+r_ducken)/2))/(4*pi*r_turkey*((r_turkey
+r_ducken)/2)*k_turkey);                     %[s*F/BTU]
```

Equivalent thermal resistance (using convection)

```
Rh = 1/(h*As); %[s*F/BTU]
```

Radiative properties of turkey

```
eps = 0.2;
sigma = (0.1713e-8)/3600; %[BTU/ft^2*s*R^4]
```

Equivalent thermal capacitance (using heat capacity)

```
C_stuffing = rho_stuffing*V_stuffing*c_stuffing; %[BTU/F]
C_ducken = rho_ducken*V_ducken*c_ducken; %[BTU/F]
C_turkey = rho_turkey*V_turkey*c_turkey; %[BTU/F]
```

Heat output of skewer

```
Qdot = 4*(5.213e-2); %[BTU/s]
```

Ambient temperature of oven

```
Tinf = 350 + 459.69; %[R]
```

Initial surface temperature

```
Tinit = 42 + 459.69; %[R]
```

Equivalent surface resistance

```
Rr = 1/(eps*sigma*(Tinit^2+Tinf^2)*(Tinit+Tinf)*As); %[s*R/BTU]
Reqinit = (Rh*Rr)/(Rh + Rr); %[s*R/BTU]
```

Initial conditions

Define a time step and final time to keep skewer on time step

N is the number of substeps, or 4.25 hours

```
dt = min([R1*C_stuffing, C_ducken/(1/R1 + 1/R2), C_turkey/(1/R4 + 1/
R5), 1]);
N = 15300;
steps = ceil(N/dt);
t = (dt*linspace(0,steps,steps+1));
```

The turducken begins cooking at 42 Fahrenheit

```
Tstuffing = Tinit*ones(1, steps);
Tducken = Tinit*ones(1, steps);
Tturkey = Tinit*ones(1, steps);
Tstuffingducken = Tinit*ones(1, steps);
Tduckenturkey = Tinit*ones(1, steps);
Tsurf = Tinit*ones(1, steps);
Rhrad = Reqinit*ones(1, steps);
```

Implement loop

```
for i=1:steps
    Tstuffing(i+1) = Tstuffing(i) + (dt/
C_stuffing)*((Tstuffingducken(i) - Tstuffing(i))/R1 + Qdot);

    Tducken(i+1) = Tducken(i) + (dt/C_ducken)*((Tstuffingducken(i) -
 Tducken(i))/R2 ...
    + (Tduckenturkey(i)-Tducken(i))/R3);

    Tturkey(i+1) = Tturkey(i) + (dt/C_turkey)*((Tduckenturkey(i)-
Tturkey(i))/R4 ...
    + (Tsurf(i)-Tturkey(i))/R5);

    Tstuffingducken(i+1) = (Tstuffing(i+1)/R1 + Tducken(i+1)/R2)/ ...
    (1/R1 + 1/R2);

    Tduckenturkey(i+1) = (Tducken(i+1)/R3 + Tturkey(i+1)/R4)/ ...
    (1/R3 + 1/R4);

    Tsurf(i+1) = (Tturkey(i+1)/R5 + Tinf/Rhrad(i))/ ...
    (1/R5 + 1/Rhrad(i));
```

```
    Rr = 1/(eps*sigma*(Tsurf(i+1)^2+Tinf^2)*(Tsurf(i+1)+Tinf)*As);
    Rhrad(i+1) = (Rh*Rr)/(Rh + Rr);

end
```

Set cooked and burned lines. These are just lines drawn on the graph. One at the temperature when the bird is considered 'cooked'. The other is the temperature when the the bird is considered 'burned'.

```
cooked = 185*ones(1,15301);
burned = 265*ones(1,15301);
```

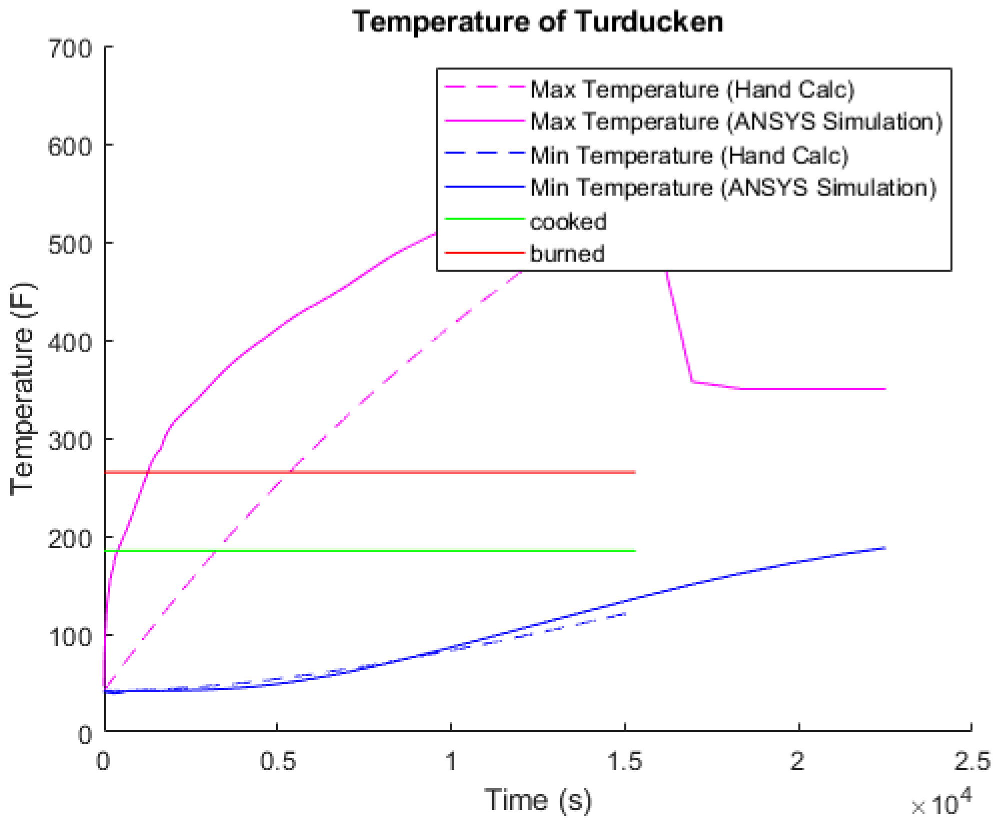
Read temperature from ANSYS

```
T = readtable("max_temps.xlsx");
A = table2array(T);
```

Plot temperatures vs. time up to when skewers turn off along with cooked and burned lines

```
figure
hold on
stuffing = plot(t,Tstuffing-459.69,'--m');
% plot(t,Tstuffingducken-459.69,'-m')
ducken = plot(t,Tducken-459.69,'--b');
% plot(t,Tduckenturkey-459.69,'-b')
% turkey = plot(t,Tturkey-459.69,'-k');
% plot(t,Tsurf-459.69,'-k');
ansysmin = plot(A(:,2),A(:,3),'-b');
ansysmax = plot(A(:,2),A(:,4),'-m');
cooked = plot(t,cooked,'-g');
burned = plot(t,burned,'-r');
% legend([stuffing ducken turkey cooked
 burned] ,'Stuffing','Ducken','Turkey','cooked','burned',
 'AutoUpdate', 'off')
legend([stuffing ansysmax ducken ansysmin cooked burned] ,'Max
 Temperature (Hand Calc)','Max Temperature (ANSYS
 Simulation)','Min Temperature (Hand Calc)','Min Temperature (ANSYS
 Simulation)','cooked','burned', 'AutoUpdate', 'off')
title 'Temperature of Turducken'
ylabel 'Temperature (F)'
xlabel 'Time (s)'
```

Warning: Column headers from the file were modified to make them valid MATLAB identifiers before creating variable names for the table. The original column headers are saved in the VariableDescriptions property. Set 'VariableNamingRule' to 'preserve' to use the original column headers as table variable names.



Skewer Off

The below functions again define the coupled ODEs for cooking. This time the skewers are not on, so there is no Qdot in the formulas.

```
dt = min([R1*C_stuffing, C_ducken/(1/R1 + 1/R2), C_turkey/(1/R4 + 1/
R5), 1]);
N = 7200;
steps = ceil(N/dt);
t = (dt*linspace(15300,steps+15300,steps+1));
```

Pre-allocation of memory below, nothing important happening.

```
Tstuffing = Tstuffing(end)*ones(1, steps);
Tducken = Tducken(end)*ones(1, steps);
Tturkey = Tturkey(end)*ones(1, steps);
Tstuffingducken = Tstuffingducken(end)*ones(1, steps);
Tduckenturkey = Tduckenturkey(end)*ones(1, steps);
Tsurf = Tsurf(end)*ones(1, steps);
Rhrad = Rhrad(end)*ones(1, steps);
```

Implementing loop

```
for i=1:steps
    Tstuffing(i+1) = Tstuffing(i) + (dt/
C_stuffing)*((Tstuffingducken(i) - Tstuffing(i))/R1);
```

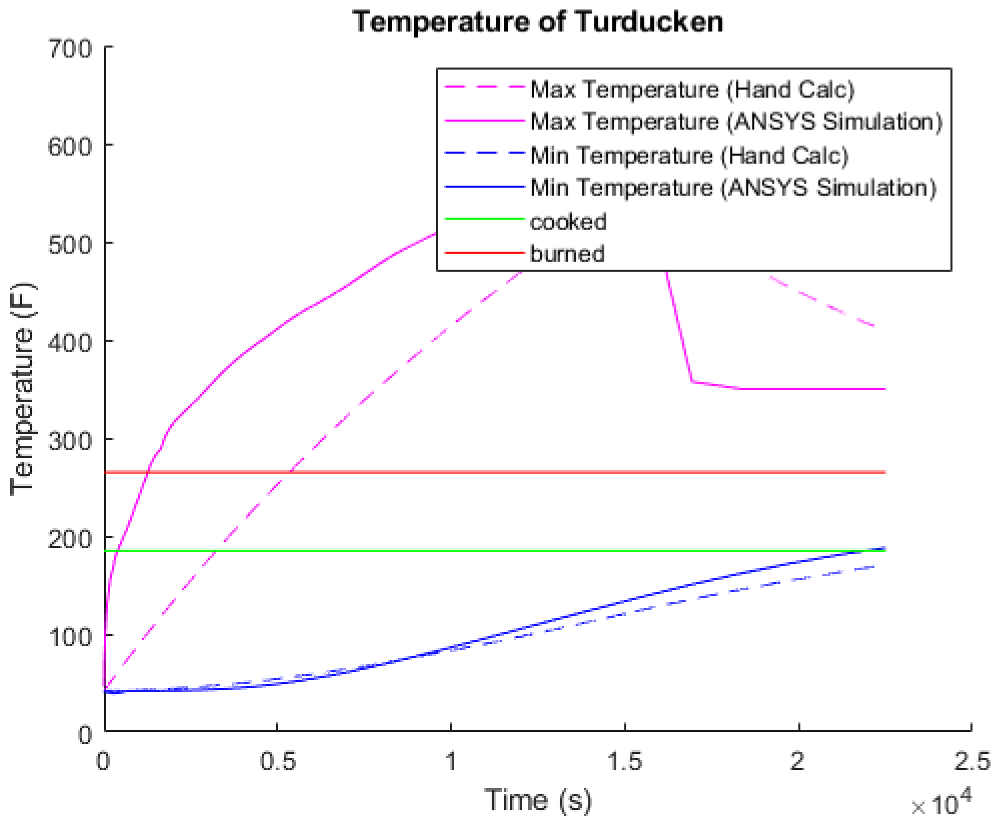
```
    Tducken(i+1) = Tducken(i) + (dt/C_ducken)*((Tstuffingducken(i) -
 Tducken(i))/R2 ...
    + (Tduckenturkey(i)-Tducken(i))/R3);

    Tturkey(i+1) = Tturkey(i) + (dt/C_turkey)*((Tduckenturkey(i)-
Tturkey(i))/R4 ...
    + (Tsurf(i)-Tturkey(i))/R5);

    Tstuffingducken(i+1) = (Tstuffing(i+1)/R1 + Tducken(i+1)/R2)/ ...
    (1/R1 + 1/R2);

    Tduckenturkey(i+1) = (Tducken(i+1)/R3 + Tturkey(i+1)/R4)/ ...
    (1/R3 + 1/R4);

    Tsurf(i+1) = (Tturkey(i+1)/R5 + Tinf/Rhrad(i))/ ...
    (1/R5 + 1/Rhrad(i));

    Rr = 1/(eps*sigma*(Tsurf(i+1)^2+Tinf^2)*(Tsurf(i+1)+Tinf)*As);
    Rhrad(i+1) = (Rh*Rr)/(Rh + Rr);
end
```

These below are continuations of the cooked/burned lines drawn earlier. They plot as a continuous line, but are coded separately so that we can make adjustments to the two halves of the code independantly

```
cooked = 185*ones(1,7201);
burned = 265*ones(1,7201);
```

plot commands again below

```
plot(t,Tstuffing-459.69,'--m');
% plot(t,Tstuffingducken-459.69,'--m')
plot(t,Tducken-459.69,'--b');
% plot(t,Tduckenturkey-459.69,'--b')
% plot(t,Tturkey-459.69,'--k');
% plot(t,Tsurf-459.69,'--k');
plot(t,cooked,'-g')
plot(t,burned,'-r')
hold off
```


Temperature of Turducken
Max Temperature (Hand Calc)
Max Temperature (ANSYS Simulation)
Min Temperature (Hand Calc)
Min Temperature (ANSYS Simulation)
cooked
burned
Temperature (F)
Time (s)
×10^4
0
100
200
300
400
500
600
700
0
0.5
1
1.5
2
2.5

Contents

Appendix E: Burned Percentage MATLAB Script

```
clearvars, close all, clc
```

Importing the ANSYS model into APDL, we can generate element lists of nodes and volumes. We can also generate node lists of temperatures. This script takes the nodal temperatures and calculates the mean element temperature. An element is considered burned if its mean temperature if above 265 F.

Read Input Spreadsheets

This section of code imports data from an excel file created to hold the data from APDL of element nodes, volumes, and nodal temperatures.

The section below is used to define vectors that we use as 'buckets' to hold the long list of information we have to process

- Element_nums holds the element number
- Element_vols holds the corresponding volume of the element
- Nodes holds the list of node numbers
- NodeTemp_Off holds the temperature of the nodes when the skewer is turned off. Since some elements go above and then sink below the burned qualification, we have to choose two points in time to look at. The first point in time is the moment we turn the skewer off, which is the peak temperature for many elements. The second point in time is the final temperature
- NodeTemp_Final holds the temperature of the nodes at the end of the cooking process
- Element_nodes holds all the nodes associated with the list of elements held in Element_nums
- Element_props holds the material IDs for the elements held in Element_nums

```
Element_nums = xlsread('Burned Percentage.xlsx',1,'A:A');
Element_vols = xlsread('Burned Percentage.xlsx',1,'B:B');
Nodes = xlsread('Burned Percentage.xlsx',3,'A:A');
NodeTemp_Off = xlsread('Burned Percentage.xlsx',3,'B:B'); %skewer turned off
NodeTemp_Final = xlsread('Burned Percentage.xlsx',3,'C:C');
Element_nodes = xlsread('Burned Percentage.xlsx',2,'B:I');
Element_props = xlsread('Burned Percentage.xlsx',2,'J:J');
```

The section below is the pre-allocation of memory to reduce solve time.

```
Element_temp_Off = zeros(length(Element_nums),8);
Element_temp_Final = zeros(length(Element_nums),8);
Element_temp_off_ave = zeros(length(Element_nums),1);
Element_temp_final_ave = zeros(length(Element_nums),1);
Element_density = zeros(length(Element_vols),1);
```

There are two nested for loops below. They step through each element and then in each element they step between the eight node numbers we have for each element. This is a list of 487182 rows and 8 columns. Due the inefficiencies of for loops in MATLAB, this implementation results in a long solve time.

The loops step through each node for each element, take the nodal temperatures and places them in the bucket we have created for the element's nodal temps called Element_final_temps. We then take the average of the nodal temps and place a final element temperature in the bucket we have made for the element temps called Element_temp_final_ave. Along side each of these calculations, a density according to the material property is calculated.

```
for i = 1:length(Element_nums)
    for j = 1:8
        ind = find(Element_nodes(i,j) == Nodes);
        if ind ~= 91643;
        Element_temp_Off(i,j) = NodeTemp_Off(ind);
        Element_temp_Final(i,j) = NodeTemp_Final(ind);
        end
    end
    Element_temp_off_ave(i) = mean(Element_temp_Off(i,:));
    Element_temp_final_ave(i) = mean(Element_temp_Final(i,:));
    if Element_props(i) == 1 | 10 | 12
        Element_density(i) = 2.0451;
    end
    if Element_props(i) == 3
        Element_density(i) = 1.8742;
    end
end
```

Total_vol is the sum of all of the element volumes Total_mass is the summ of the volume of each element multiplied by its density (a calculation performed in the nested for loops)

```
Total_vol = sum(Element_vols(:,1));
Total_mass = sum(Element_vols(:,1).*(Element_density(:,1)));
```

The following loop checks if the element is burned either at the time that the skewers are turned off or at the time the turducken is pulled from the oven.

```
for i = 1:length(Element_temp_final_ave)
    if Element_temp_final_ave(i) > 265 |  Element_temp_off_ave(i) > 265
        Volburned(i,1) = Element_vols(i);
        Massburned(i,1) = Element_vols(i)*Element_density(i);
    end
end
```

The final two calculations yield out the percentage of the turducken that the process burnt by volume and mass

```
Percent_burned_vol = sum(Volburned(:,1))/Total_vol
Percent_burned_mass = sum(Massburned(:,1))/Total_mass
```

```
Percent_burned_vol =

    0.3898


Percent_burned_mass =

    0.3957
```